

Multi-physics and multi-scale simulations for hydropower and geo-energy

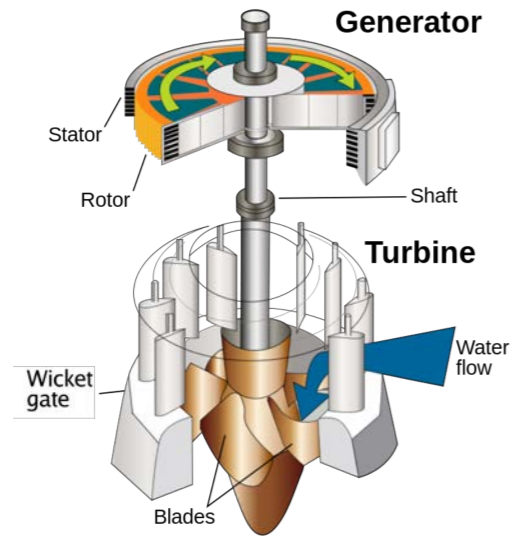
Patrick Zulian, Maria Nestola, Marco Favino, Cyrill von Planta, Rolf Krause
Collaboration with: Jürg Hunziker, Klaus Holliger,
Xiaoqing Chen, Daniel Vogler, Martin Saar

Institute of Computational Science, Università della Svizzera italiana
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ETHZ, GEG

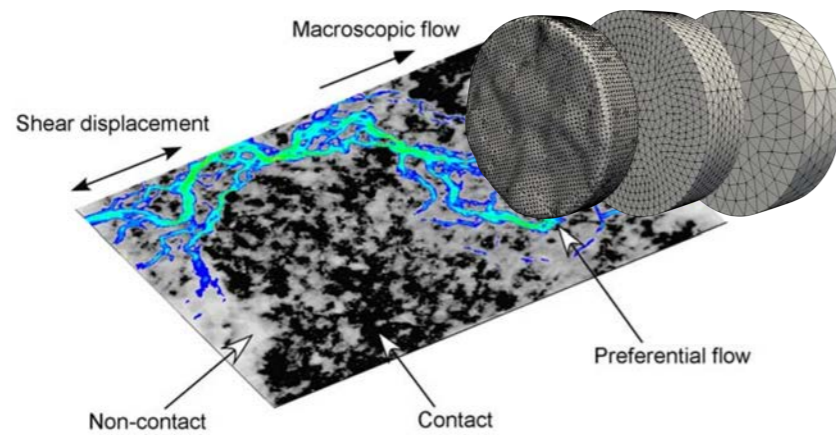
September 13, 2018, Horw, Lucerne



Computational energy



Water turbines



Computational Geophysics



SWISS COMPETENCE CENTER for ENERGY RESEARCH
SUPPLY of ELECTRICITY

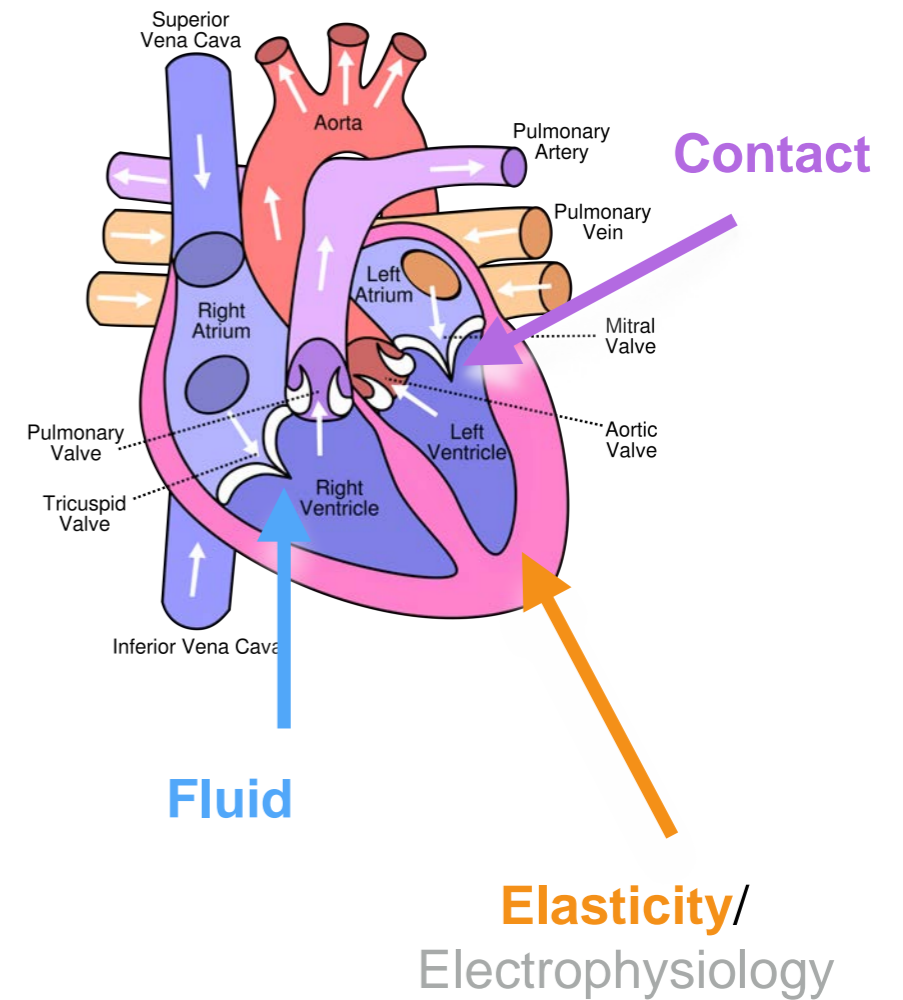


High-performance computing

Usability
+
Flexibility!

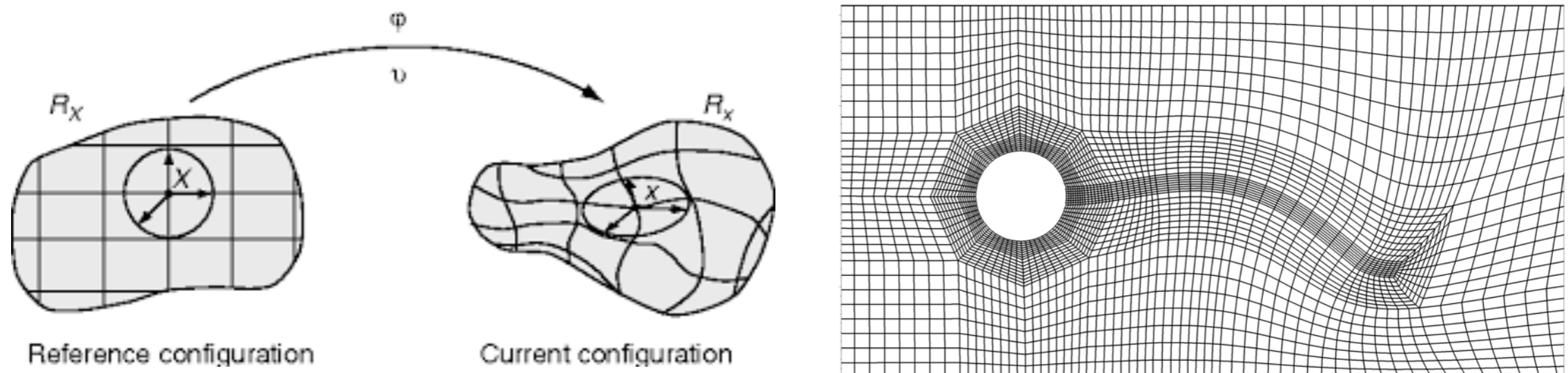
Other applications

Simulation of the heart



Boundary fitted method

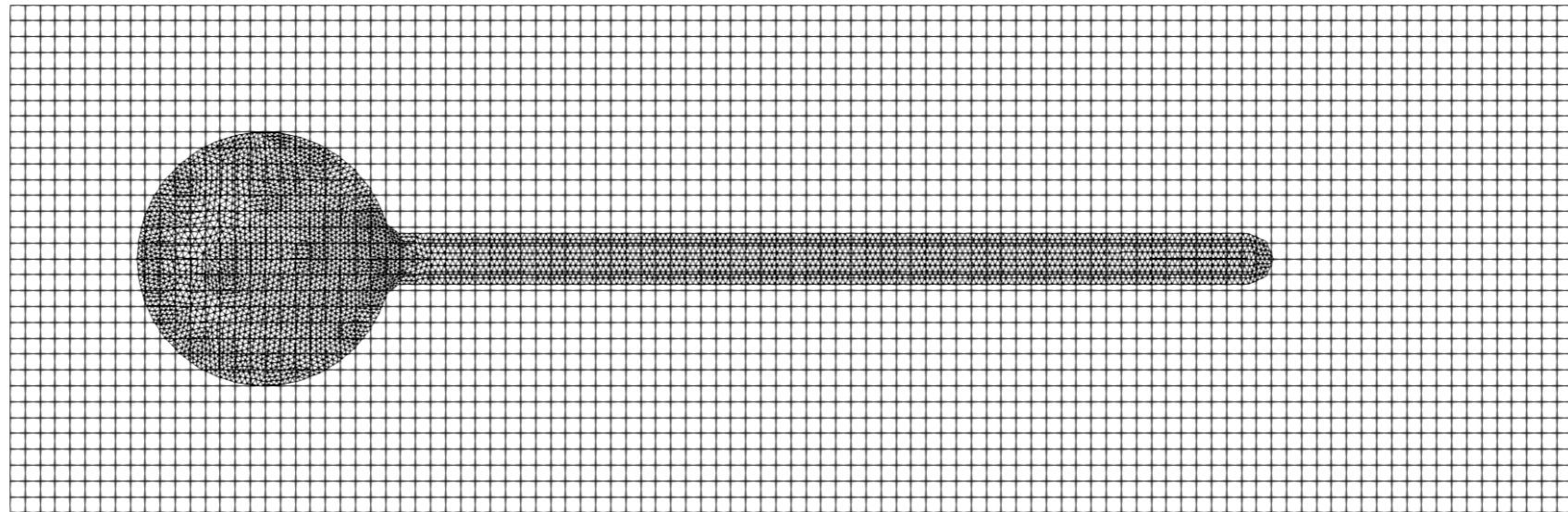
- Arbitrary Lagrangian Eulerian (ALE)
- Fluid mesh moves with the solid mesh
- **Accurate** results at FSI interface
- Large displacements → **Distorted fluid grid** → Numerical stability and accuracy
- Meshing, Re-meshing → artificial diffusion



Literature: Jianhai, Dapeng, and Shengquan 1996. Morsi, Yang, Wong, and Das 2007.

Fictitious domain method

- Solid phase is embedded in the fluid phase
- **Fixed grid** → Eulerian formulation
- **Greater** grid **resolution** necessary for reproducing similar results
- Different discretizations and software can be easily used together (e.g., Finite Difference and FEM)



Literature:

1) A fictitious domain/mortar element method for fluid–structure interaction, Baaijens, 2001.

2) A mortar approach for Fluid–Structure interaction problems: Immersed strategies for deformable and rigid bodies, Hesch et al.

Find $(\mathbf{u}_f, p_f; \boldsymbol{\eta}_s, p_s; \boldsymbol{\lambda}) \in (\mathbf{V}_f \times Q_f \times \mathbf{V}_s \times Q_s \times \mathbf{L})$ such that

$$\int_{\Omega_f} \rho_f \frac{\partial \mathbf{u}_f}{\partial t} \cdot \mathbf{v}_f dV + \int_{\Omega_f} \rho_f [(\mathbf{u}_f \cdot \nabla) \mathbf{u}_f] \cdot \mathbf{v}_f dV + \int_{\Omega_f} \boldsymbol{\sigma}_f \cdot \mathbf{v}_f dV - \int_{\mathcal{I}} \boldsymbol{\lambda} \cdot \mathbf{v}_f dV = 0$$

$$\int_{\Omega_f} q_f \nabla \cdot \mathbf{u}_f dV = 0$$

$$\int_{\mathcal{I}} \boldsymbol{\mu} \cdot \left(\frac{\partial \boldsymbol{\eta}_s}{\partial t} - \mathbf{u}_f \right) dV = 0$$

$$\int_{\hat{\Omega}_s} \hat{\rho}_s \frac{\partial^2 \hat{\boldsymbol{\eta}}_s}{\partial t^2} \cdot \hat{\mathbf{v}}_s + \int_{\hat{\Omega}_s} \hat{\mathbf{P}}(\hat{\mathbf{F}}) : \nabla \hat{\mathbf{v}}_s dV - \int_{\hat{\Omega}_s} \hat{p}_s \hat{\mathbf{J}} \hat{\mathbf{F}}^{-T} : \nabla \hat{\mathbf{v}}_s dV + \int_{\mathcal{I}} \boldsymbol{\lambda} \cdot \hat{\mathbf{v}}_s dV = 0$$

$$\int_{\hat{\Omega}_s} (\hat{\mathbf{J}} - 1) q_s dV = 0$$

for all $(\mathbf{v}_f, q_f; \mathbf{v}_s, q_s; \boldsymbol{\mu}) \in (\mathbf{V}_f \times Q_f \times \mathbf{V}_s \times Q_s \times \mathbf{L})$, where

$$\mathcal{I} = \Omega_s \cap \Omega_f$$

Article: An immersed boundary method based on the variational L^2 projection approach

M. Nestola, B. Becsek, H. Zolfaghari, P. Zulian, D. Obrist and R. Krause.

Submitted to the proceedings of the 24th International Conference of Domain Decomposition Methods, 2017

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Transfer \rightarrow

$$\int_{\mathcal{I}} \boldsymbol{\mu} \cdot \left(\frac{\partial \boldsymbol{\eta}_s}{\partial t} - \mathbf{u}_f \right) dV = 0$$

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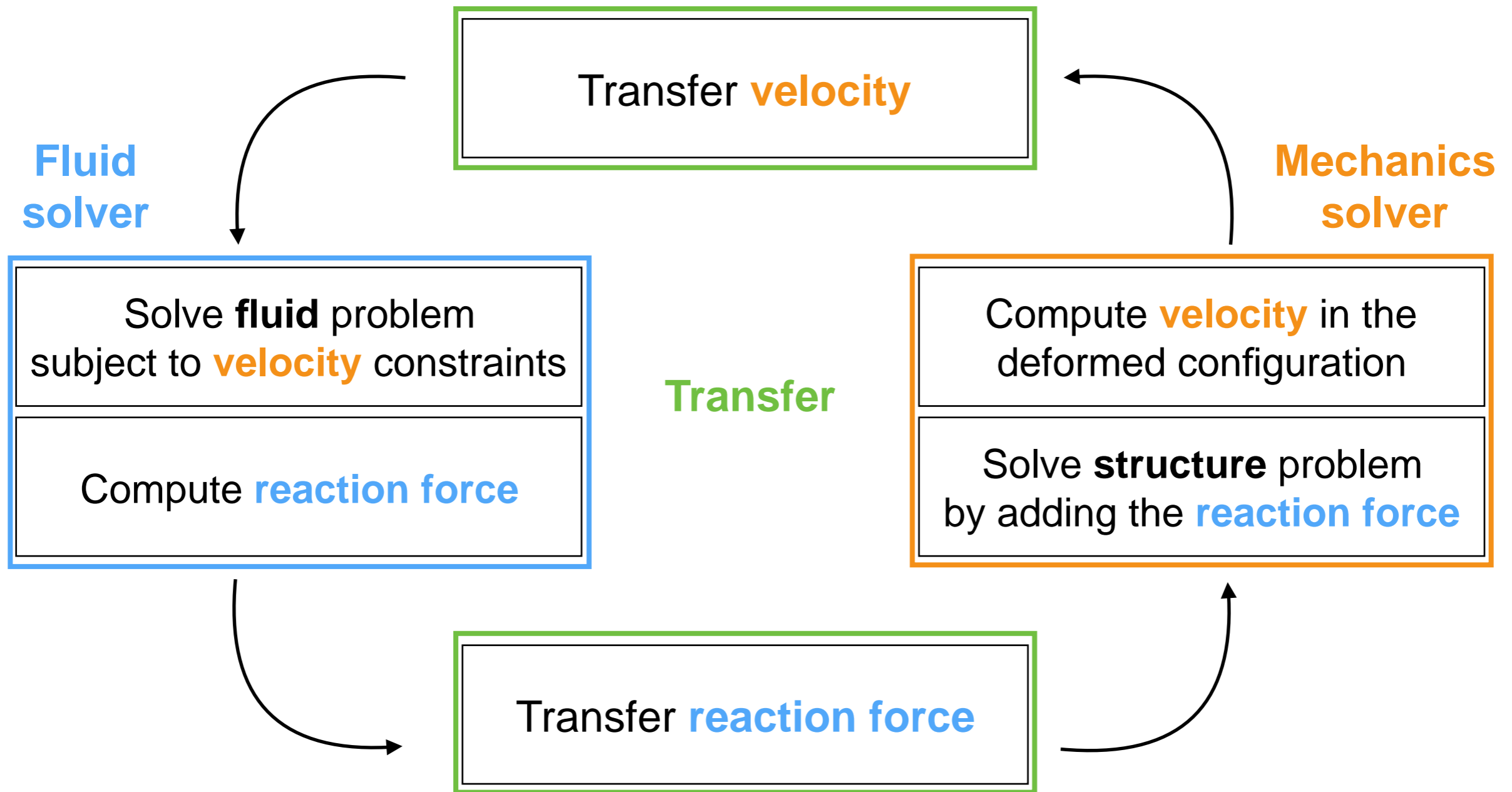
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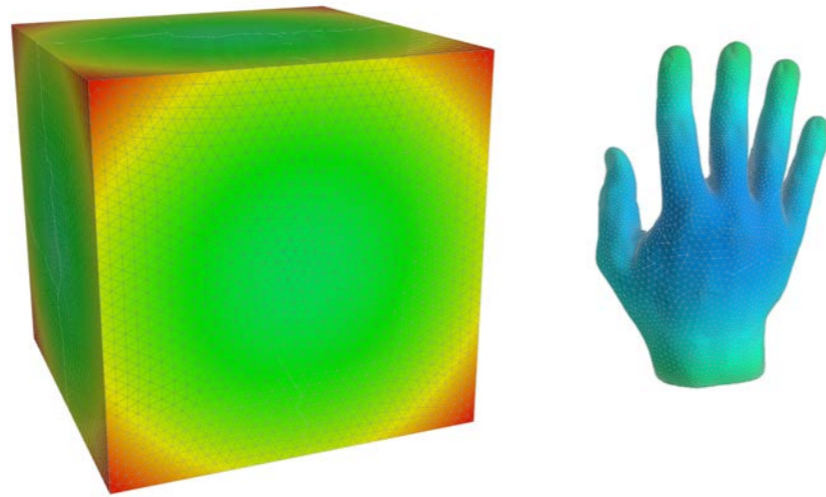
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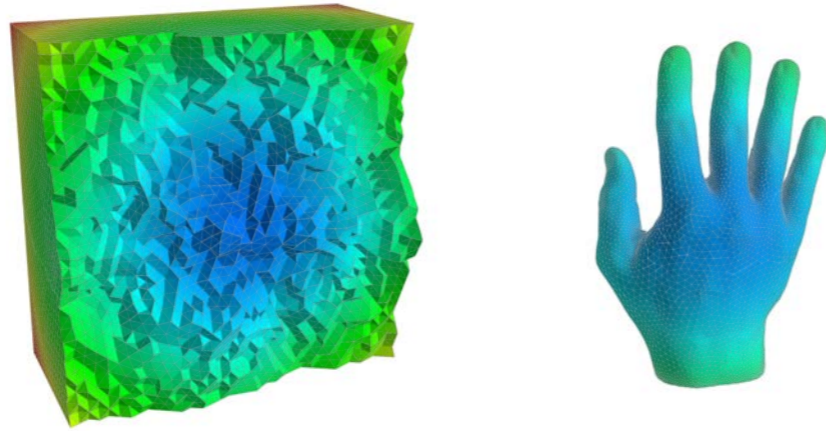
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Fixed point iteration

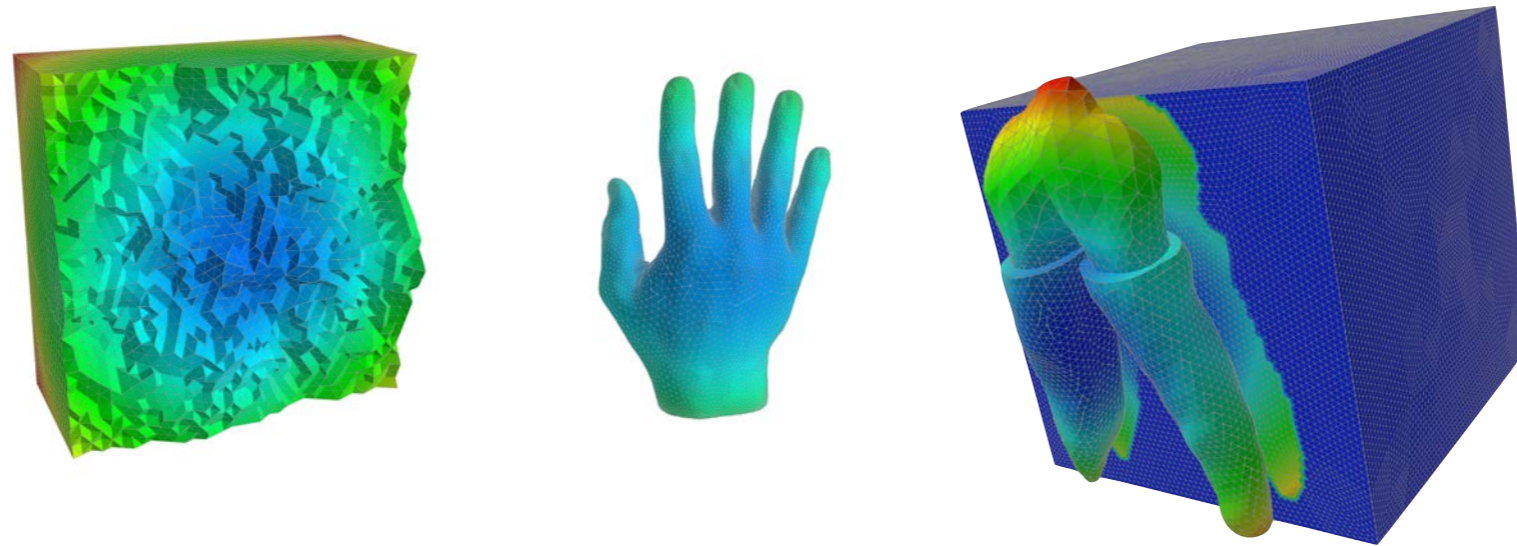




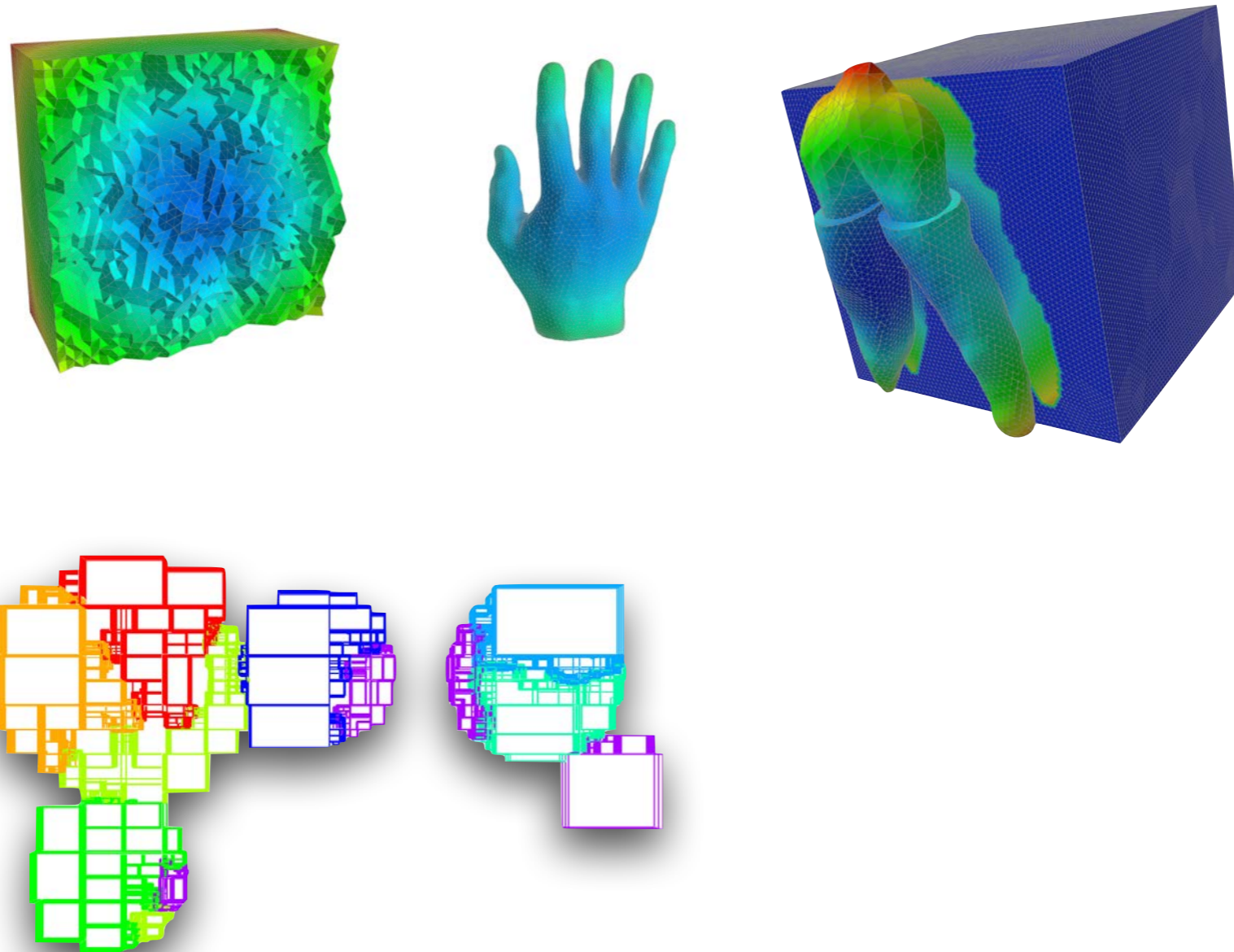
Article: **A parallel approach to the variational transfer of discrete fields between arbitrarily distributed unstructured finite element meshes**, R. Krause and P. Zulian, SIAM Journal of Scientific Computing 2016



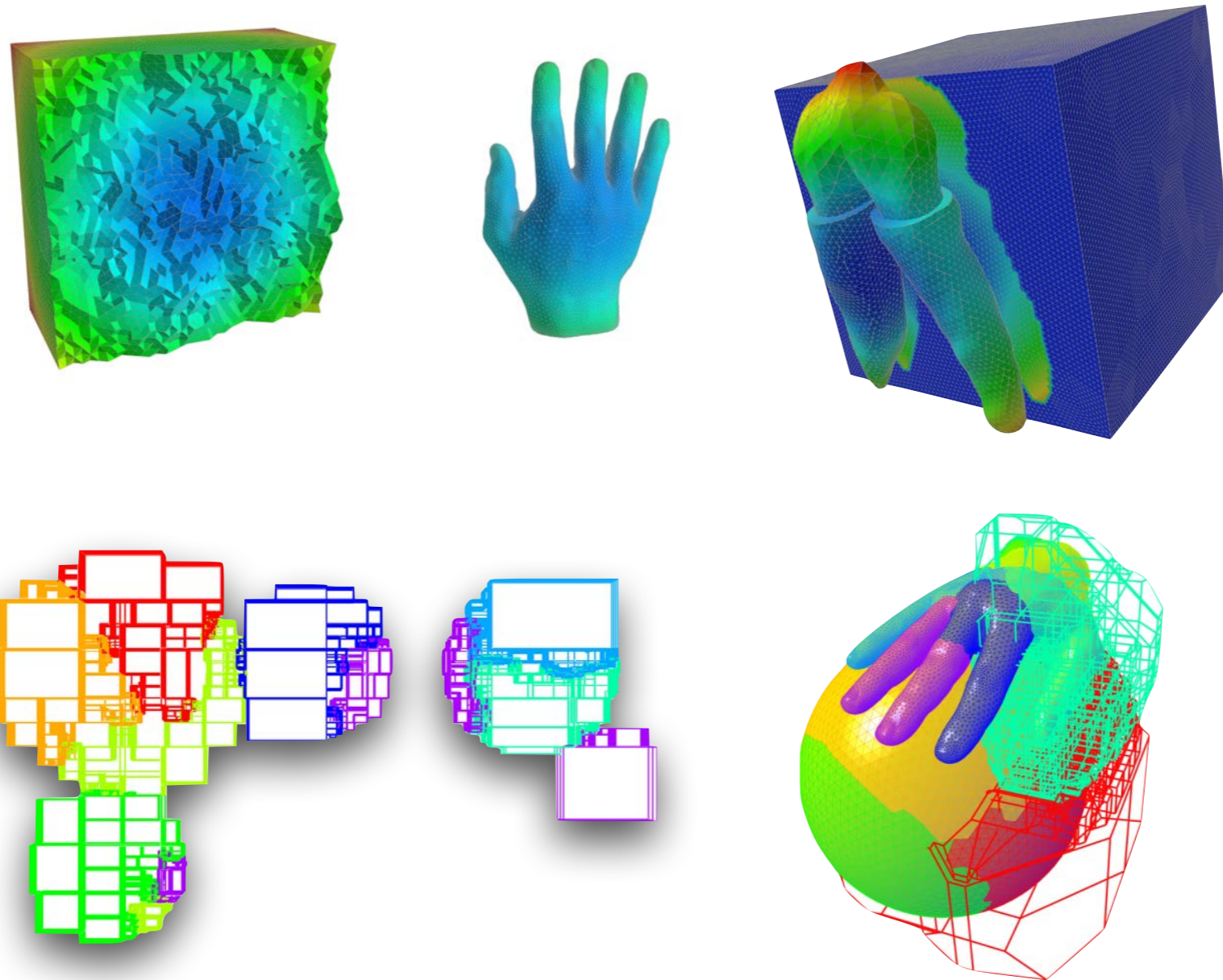
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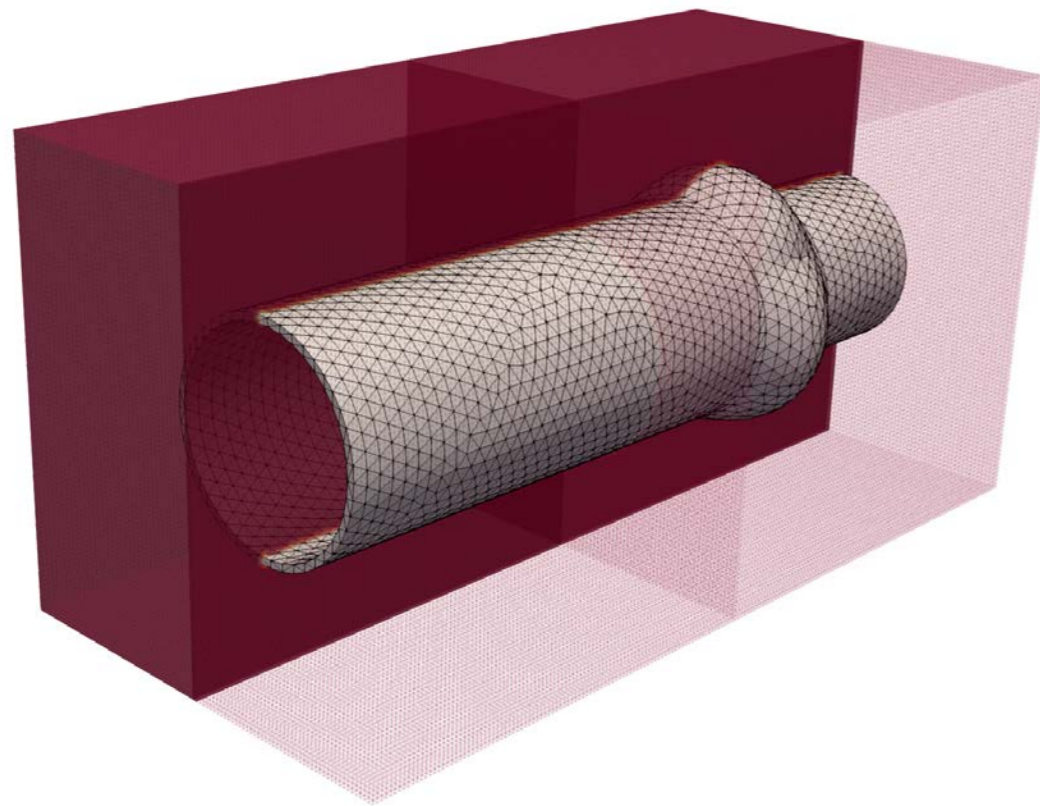
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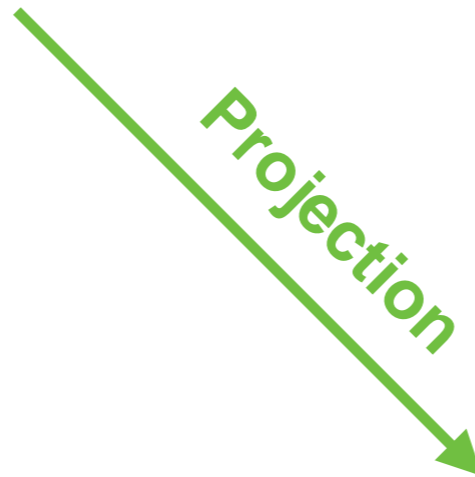
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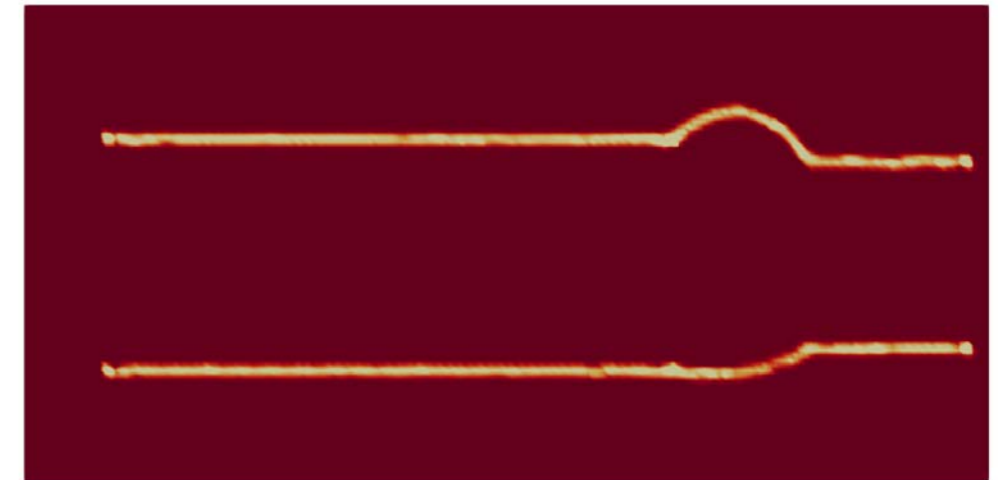
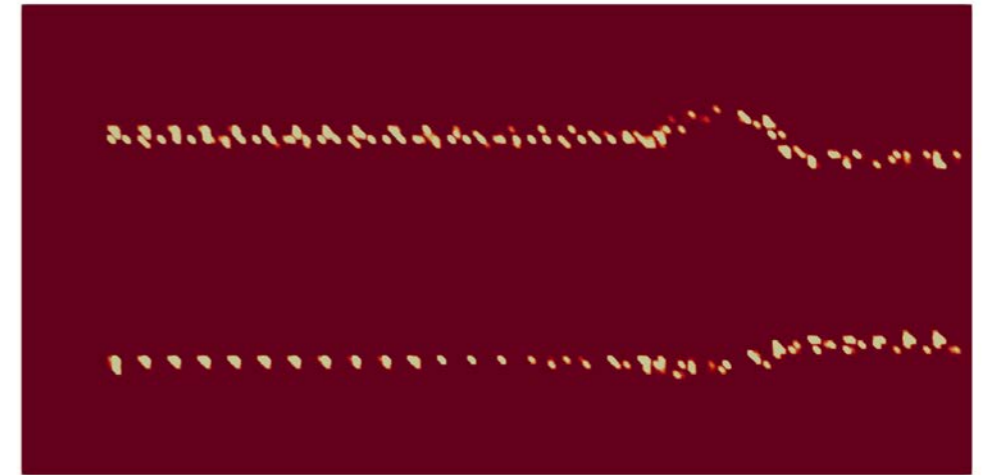
Interpolation



Projection



Adjoint



L²-projection

→ Optimal, stable, computationally expensive

Interpolation

→ Does not pass the patch test,
computationally cheaper, simpler for higher-
order FE deformations

- Definition of the L^2 -projection operator $P: V_h \rightarrow W_h$

- For $v_h \in V_h(\mathcal{T}_m)$ find $w_h = P(v_h) \in W_h(T_s)$

$$(P(v_h), \mu_h)_{L^2(I_h)} = (v_h, \mu_h)_{L^2(I_h)} \quad \forall \mu_h \in M_h$$

- Weak-equality condition

$$\int_{I_h} (v_h - P(v_h)) \mu_h \, d\mathbf{x} = \int_{I_h} (v_h - w_h) \mu_h \, d\mathbf{x} = 0 \quad \forall \mu_h \in M_h$$

Let $\text{span}\{\phi_i\}_{i \in J^V} = V_h$, $\text{span}\{\theta_j\}_{j \in J^W} = W_h$ and
 $\text{span}\{\psi_k\}_{k \in J^M} = M_h$.

We can now write $v_h = \sum_{i \in J^V} v_i \phi_i$ and $w_h = \sum_{j \in J^W} w_j \theta_j$

Dual Lagrange multipliers (Pseudo- L^2 -projection)

Literature: Wohlmuth, 1998. Dickopf and Krause 2014.

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and the node-wise contributions

$$\sum_{i \in J^V} v_i \int_{I_h} \phi_i \psi_k d\mathbf{x} = \sum_{j \in J^W} w_j \int_{I_h} \theta_j \psi_k d\mathbf{x} \quad \text{for } k \in J^M$$

$$\implies \mathbf{B}\mathbf{v} = \mathbf{D}\mathbf{w} \quad \text{with } b_{ik} = \int_{I_h} \phi_i \psi_k d\mathbf{x}, \quad d_{jk} = \int_{I_h} \theta_j \psi_k d\mathbf{x}$$

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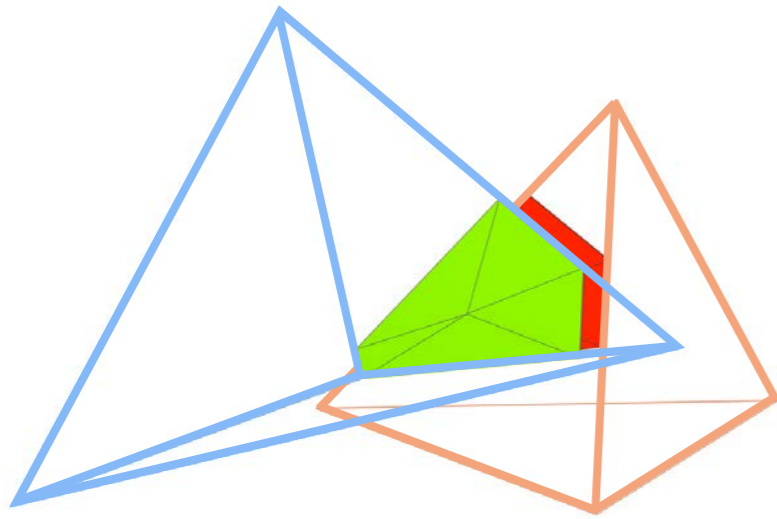
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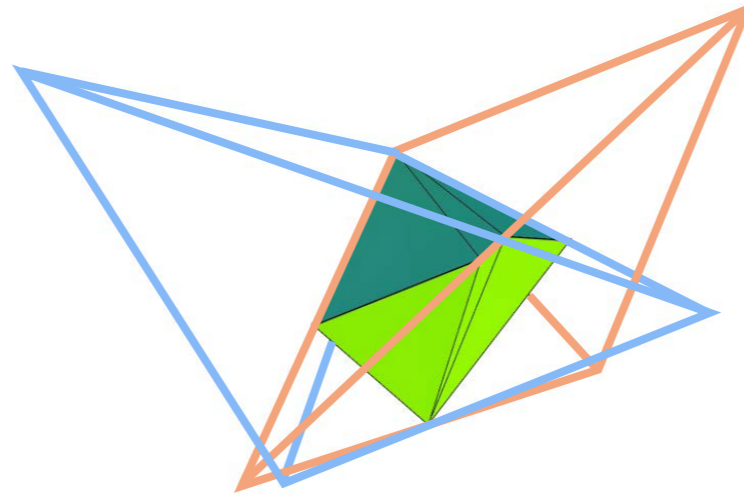
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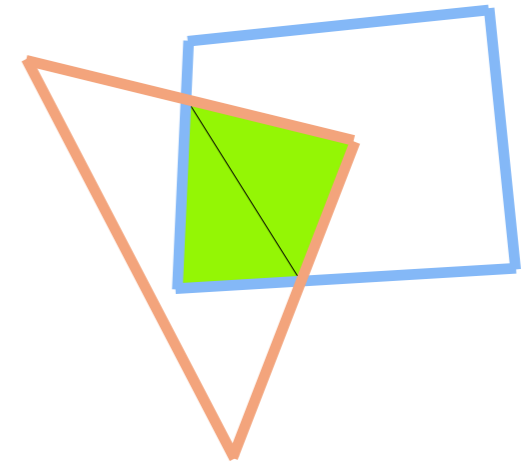
From intersection to quadrature rule



Surface normal projection

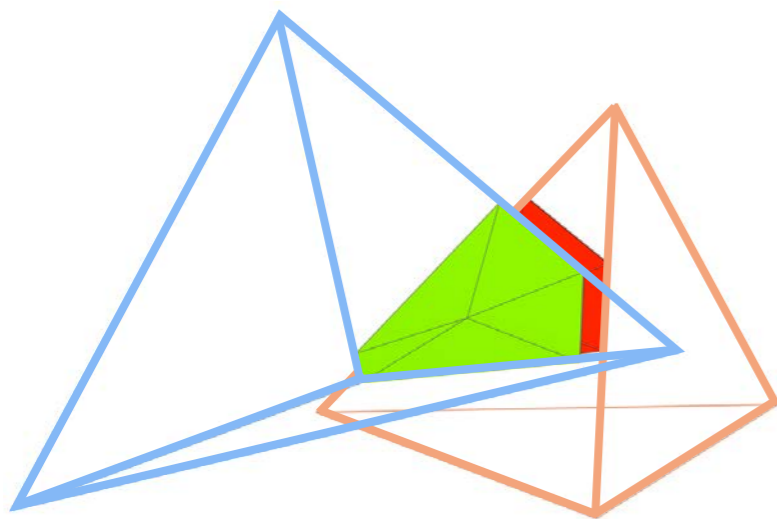


Volume intersection 3D

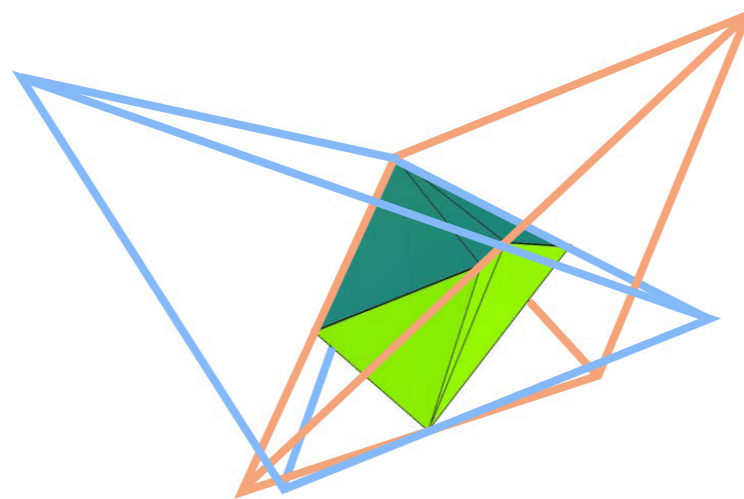


Volume intersection 2D

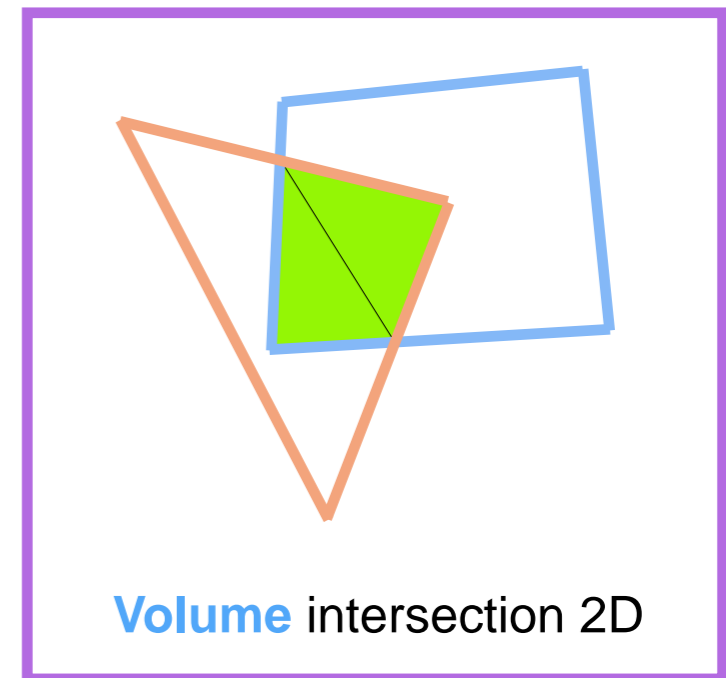
From intersection to quadrature rule



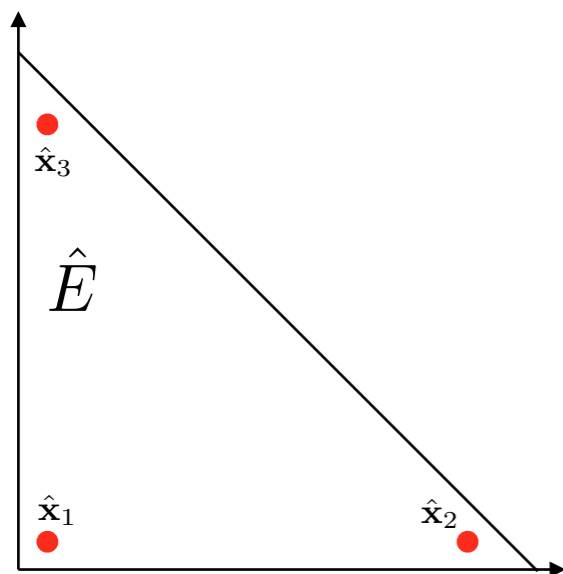
Surface normal projection



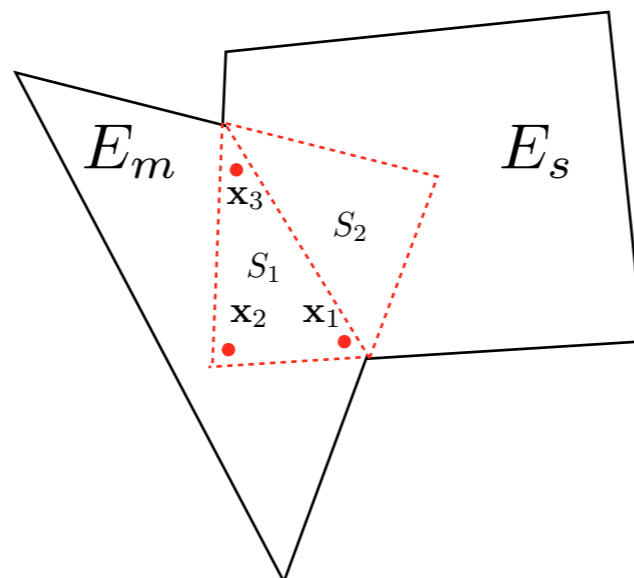
Volume intersection 3D



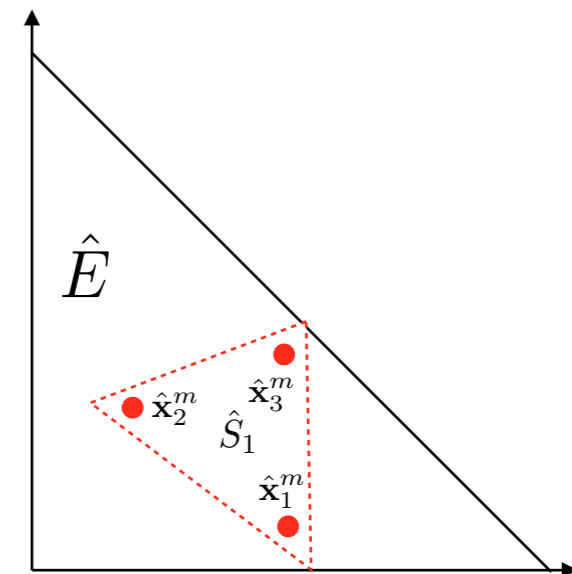
Volume intersection 2D



Reference element



Physical coordinates

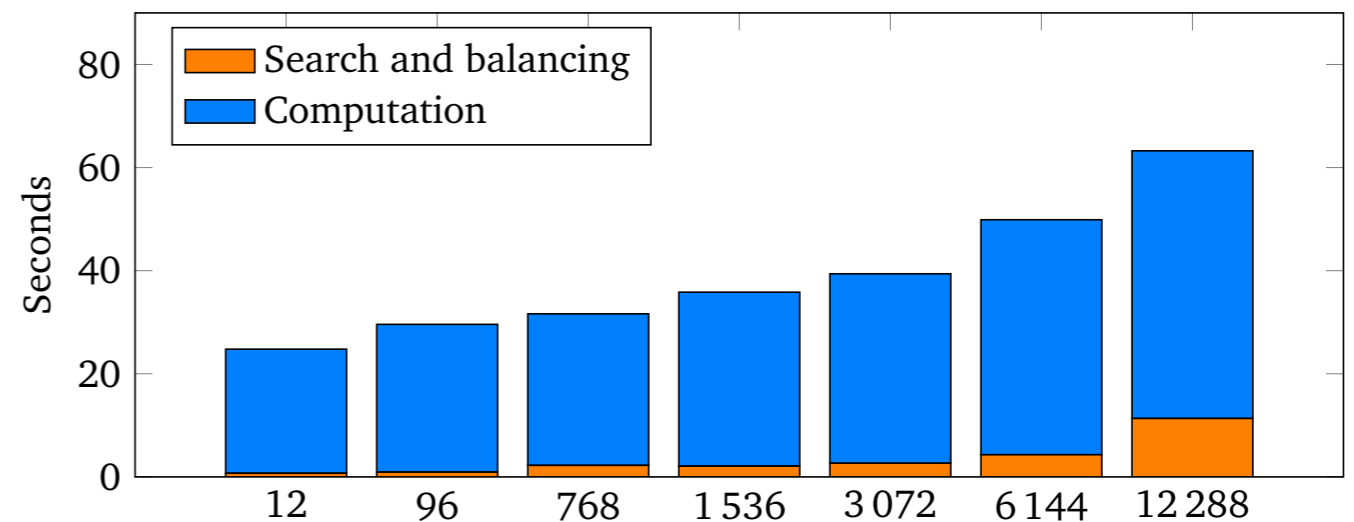
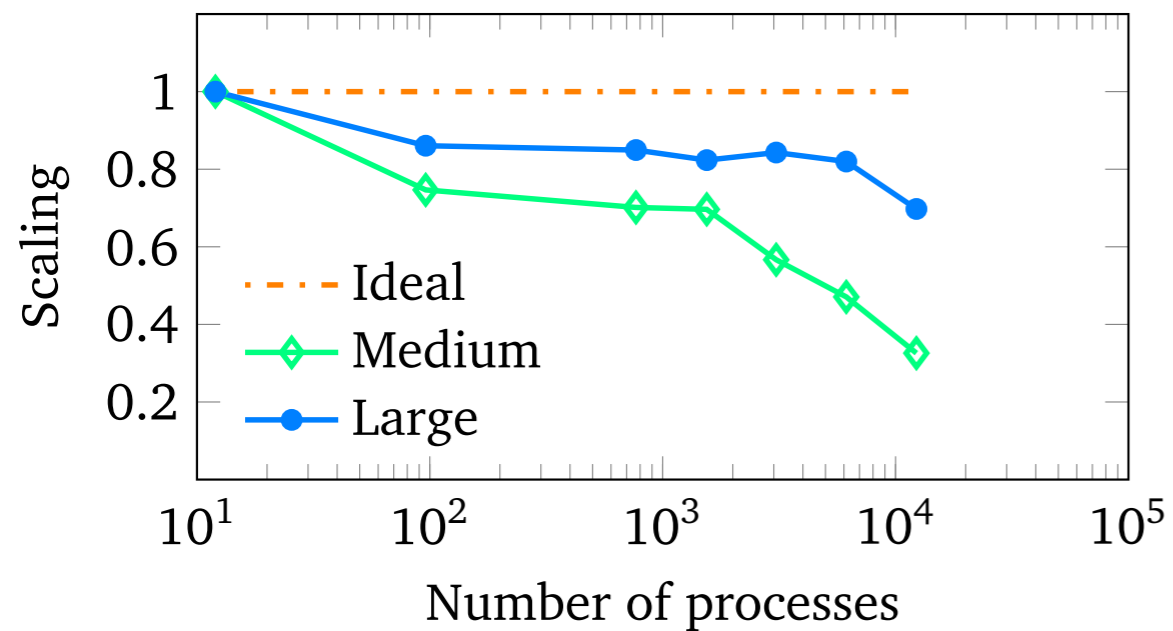
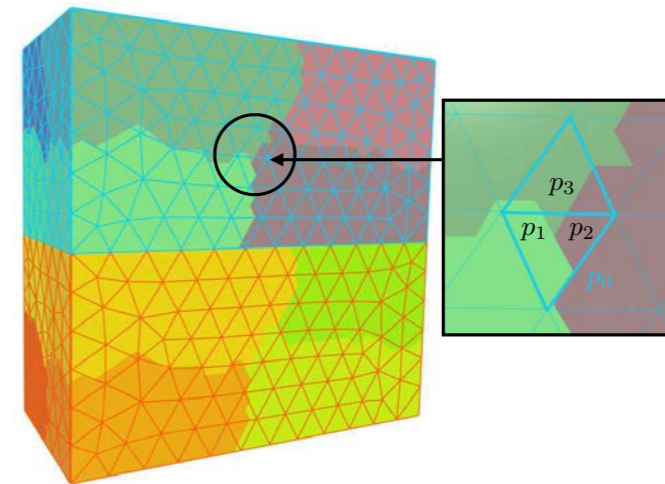


Reference element

Weak scaling (volume projections)

Experiments:

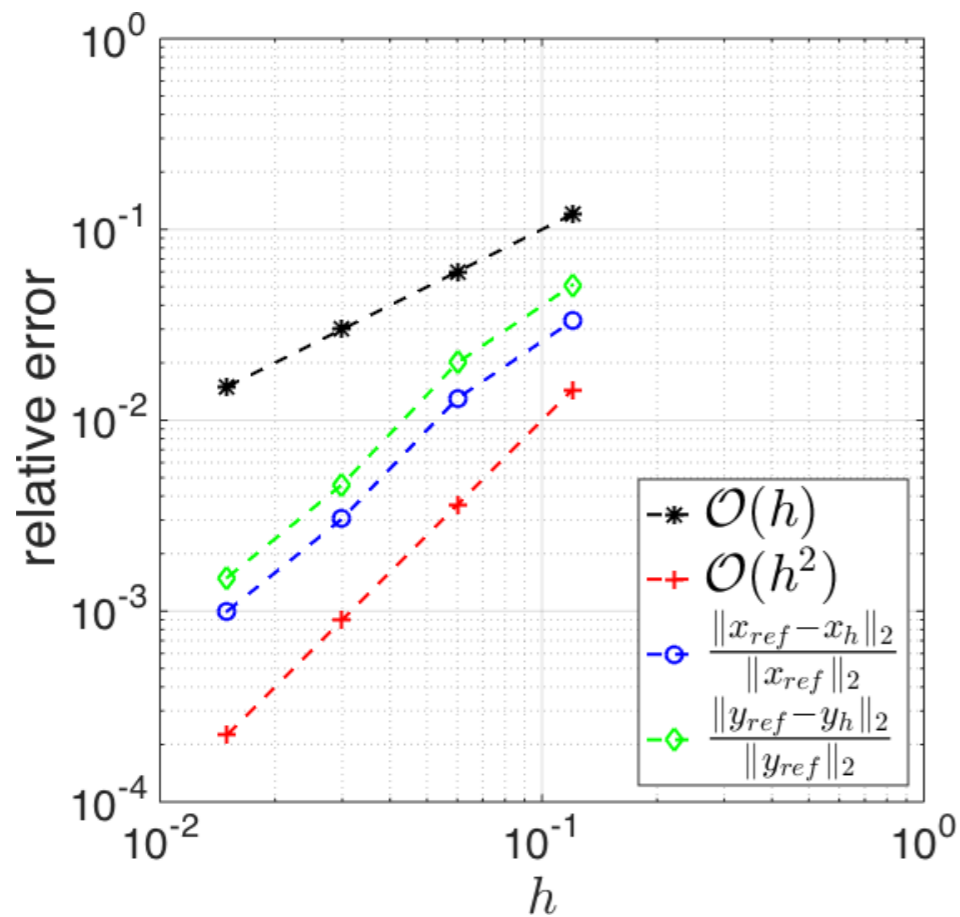
- **Small** 10 000 elements per process
- **Large** 150 000 elements per process
- Output is x4



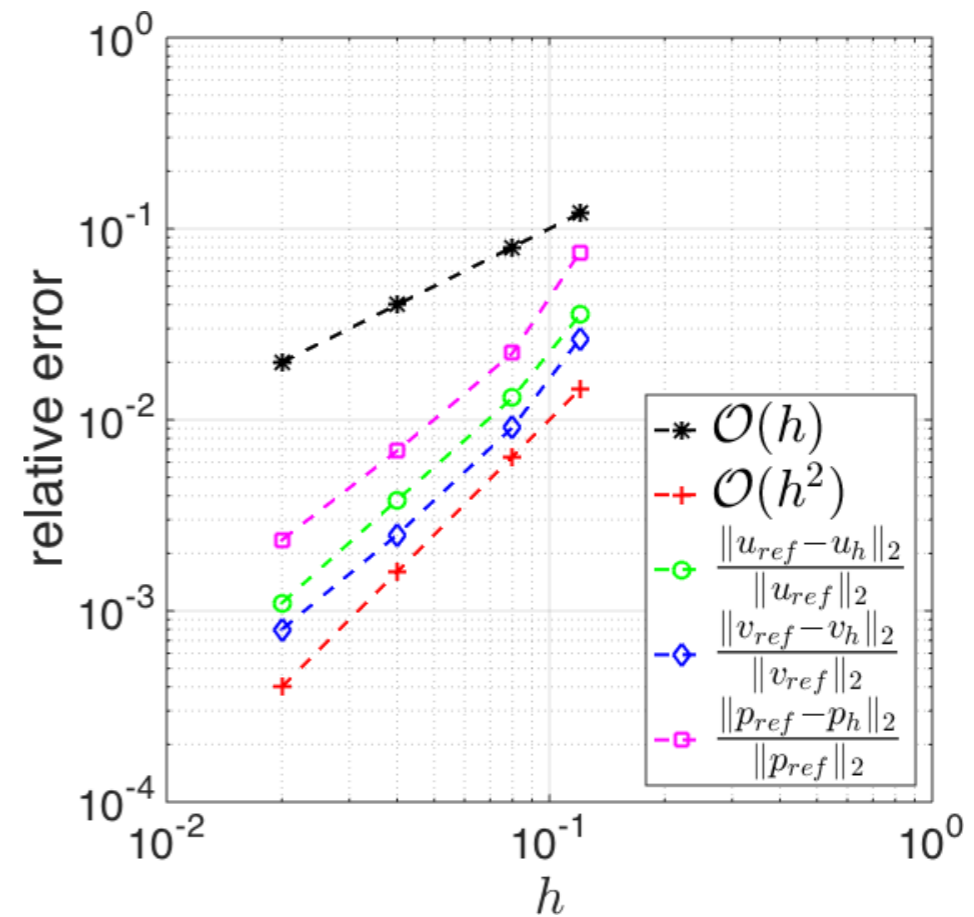
Weak scaling is measured as (time base experiment)/(time experiment)

Discretisation error

- L^2 -convergence



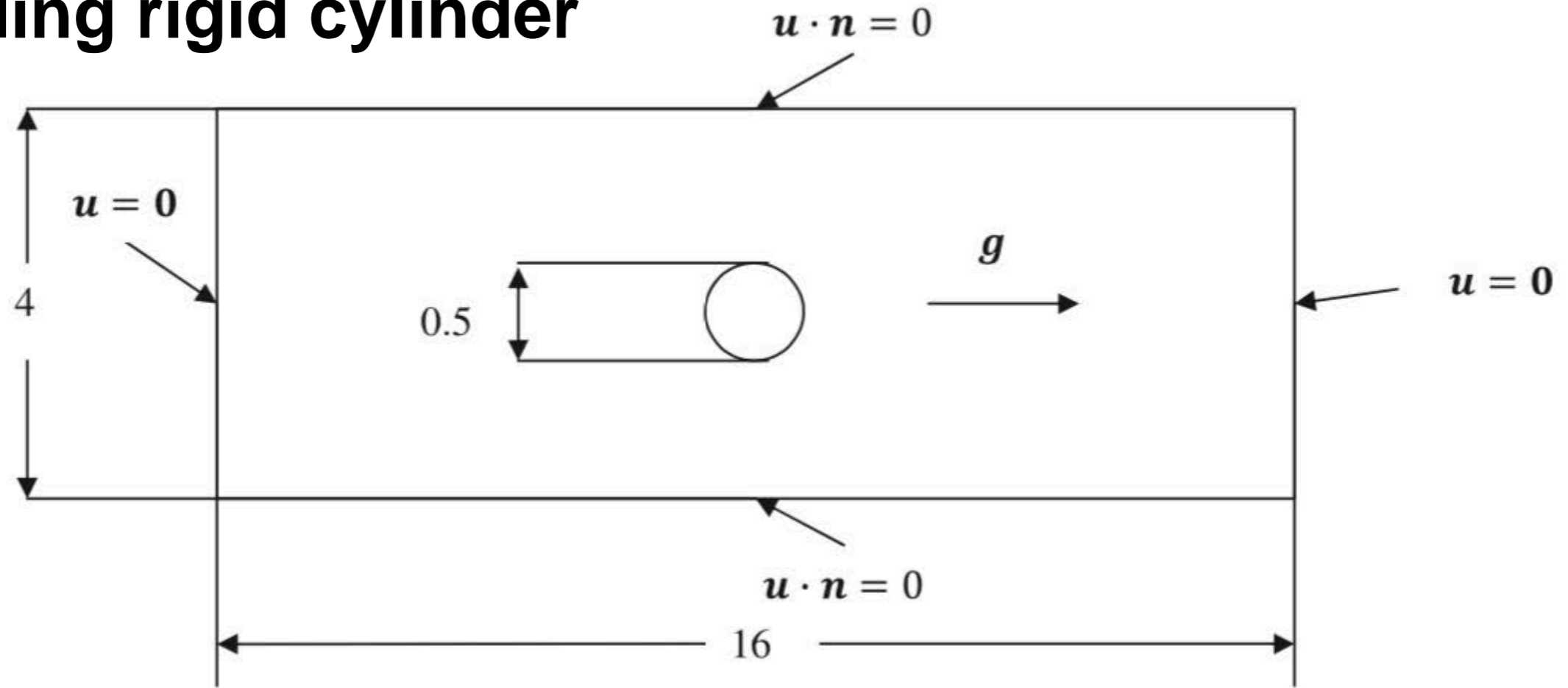
Solid displacement



Fluid velocity

Validation: FSI

Free falling rigid cylinder



Fluid density

$$\rho_f = 1.0 [g/cm^3]$$

Newtonian Fluid

$$\sigma_f = -pI + \mu_f \frac{(\nabla^T v_f + \nabla v_f)}{2}$$

$$\mu_f = 1.0 \text{ dyne}/cm^2 s$$

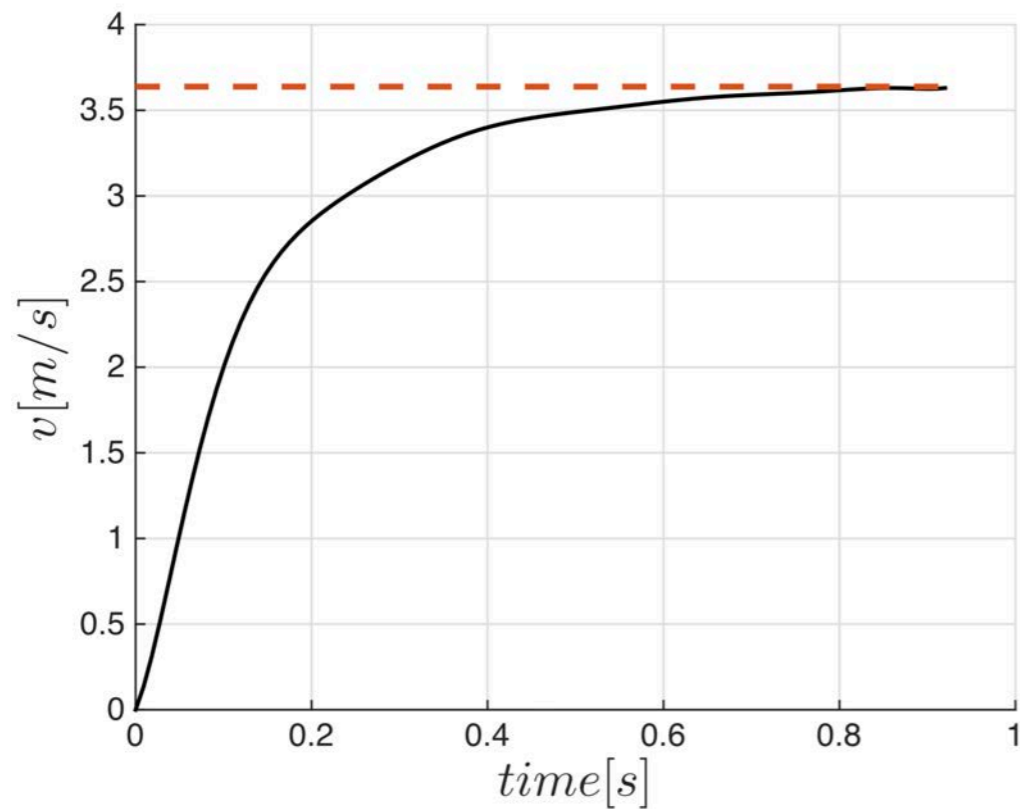
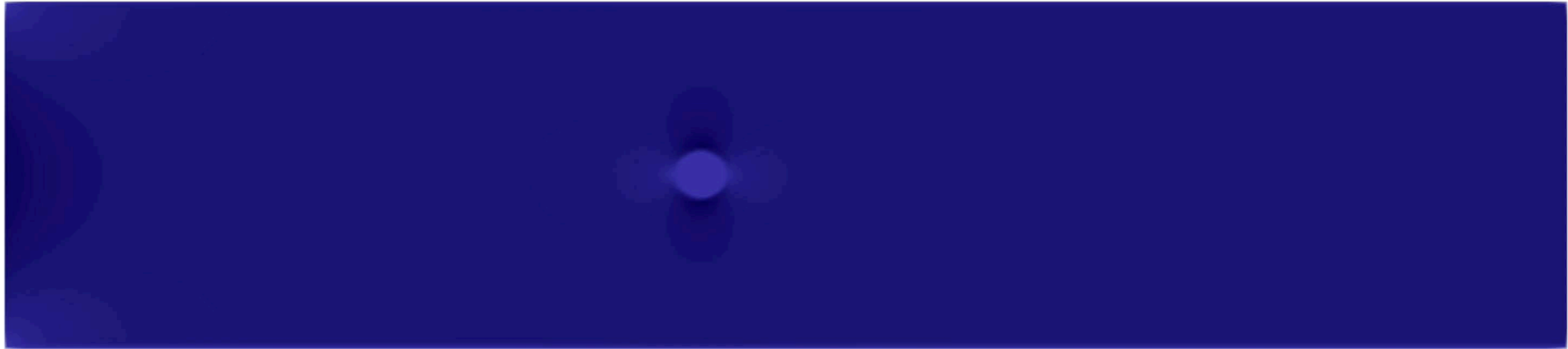
Solid density

$$\rho_s = 1.20 [g/cm^3]$$

Linear Elastic

$$\mu_s = 3846.1538 \text{ dyne}/cm^2$$

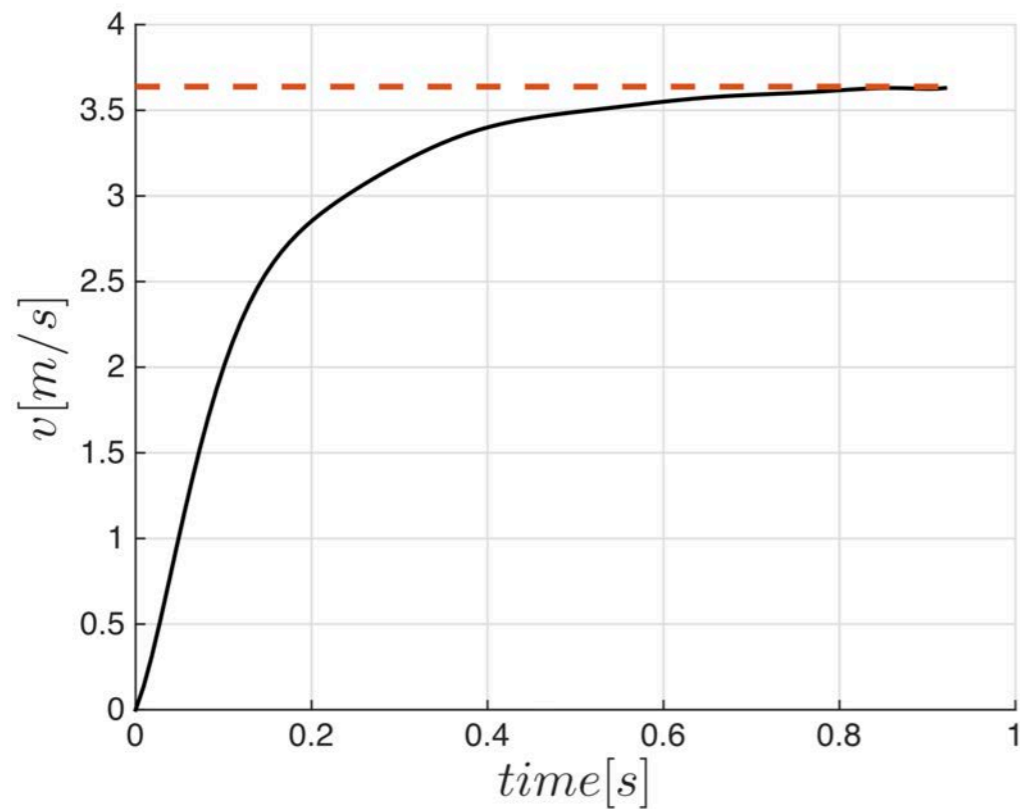
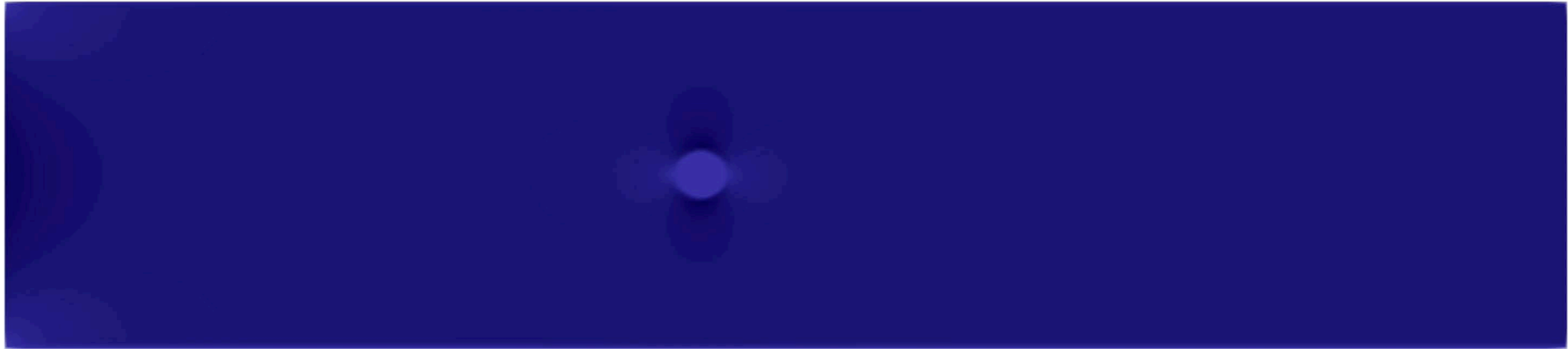
Free falling rigid cylinder



Terminal Velocity

$$\frac{(\rho_s - \rho_f)ga^2}{4\mu} \left[\ln \left(\frac{L}{a} \right) - 0.9157 + 1.7244 \left(\frac{a}{L} \right)^2 - 1.7302 \left(\frac{a}{L} \right)^4 \right]$$

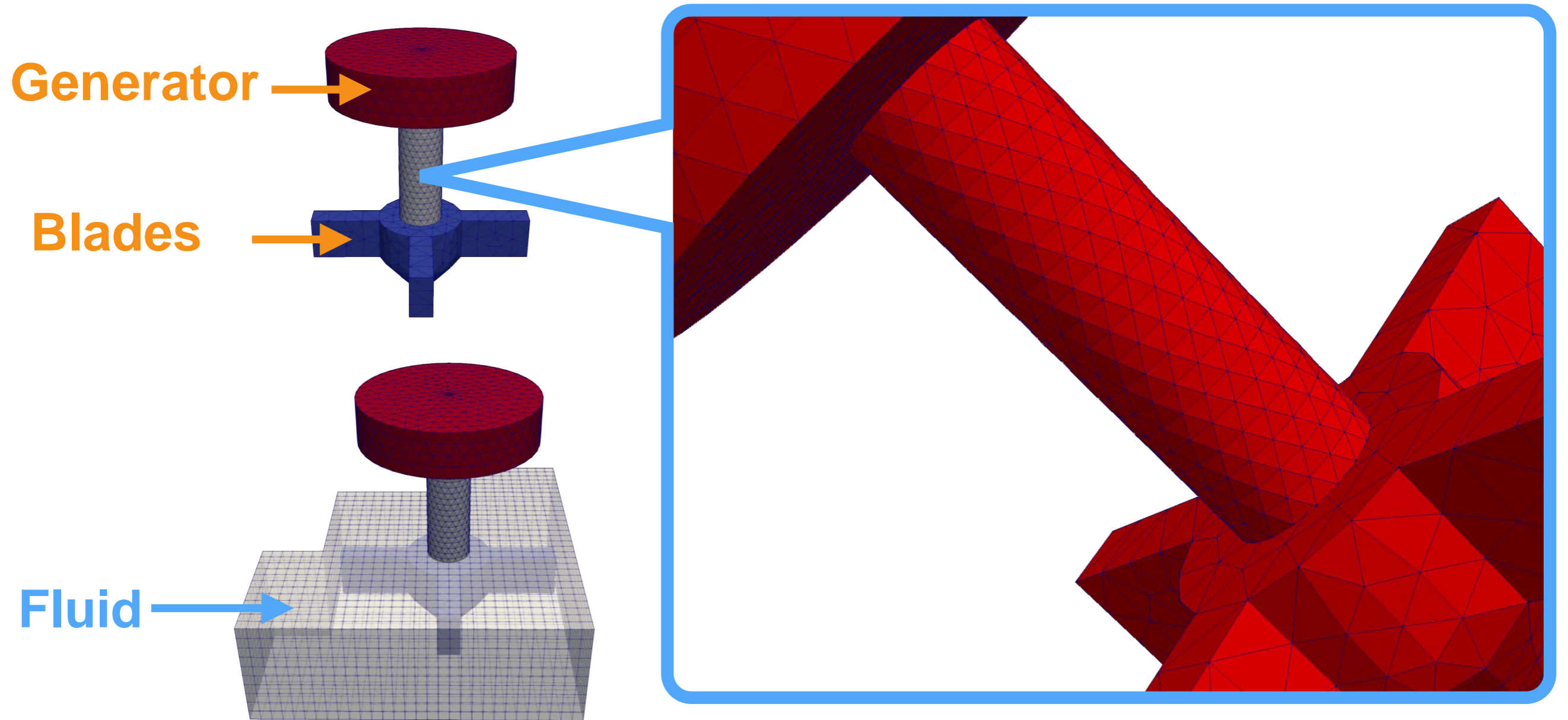
Free falling rigid cylinder



Terminal Velocity

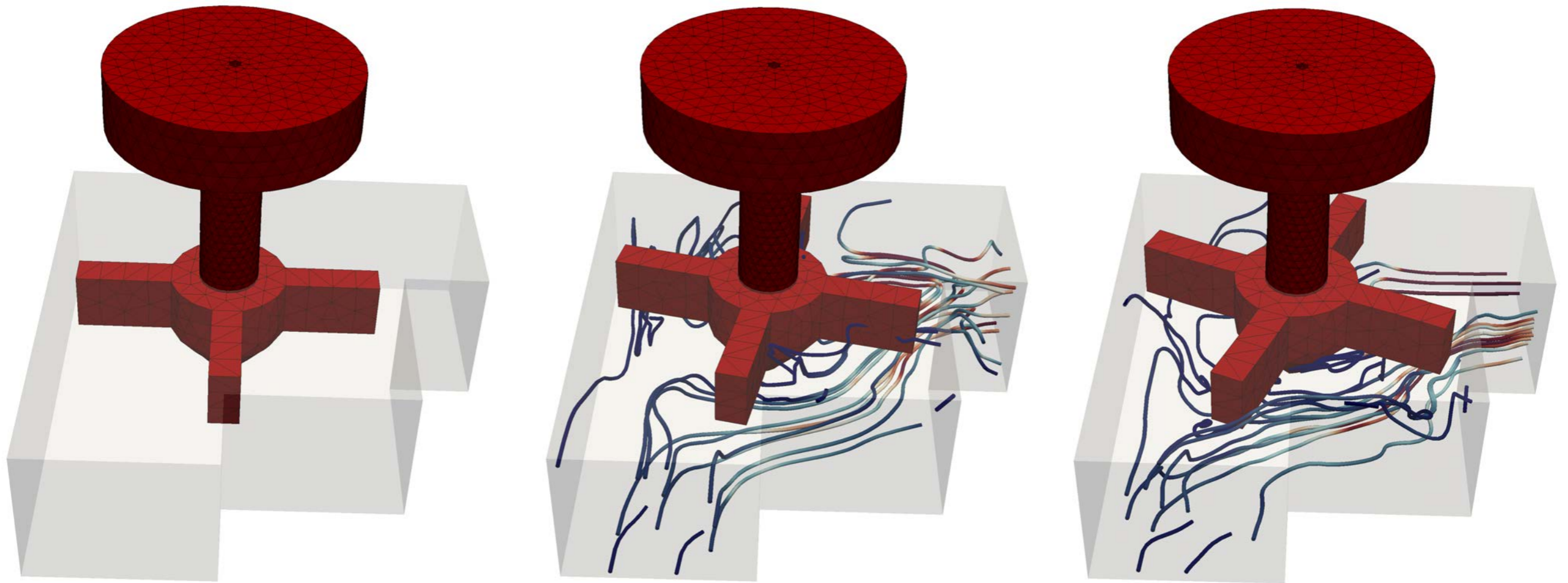
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Mesh-tying and FSI on idealised turbines (1)

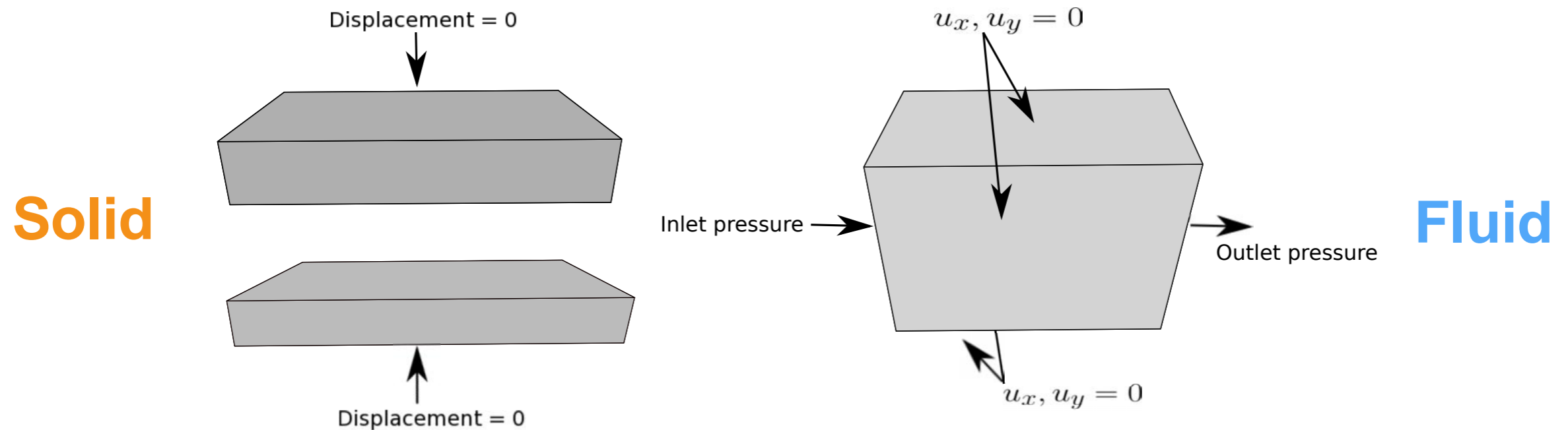
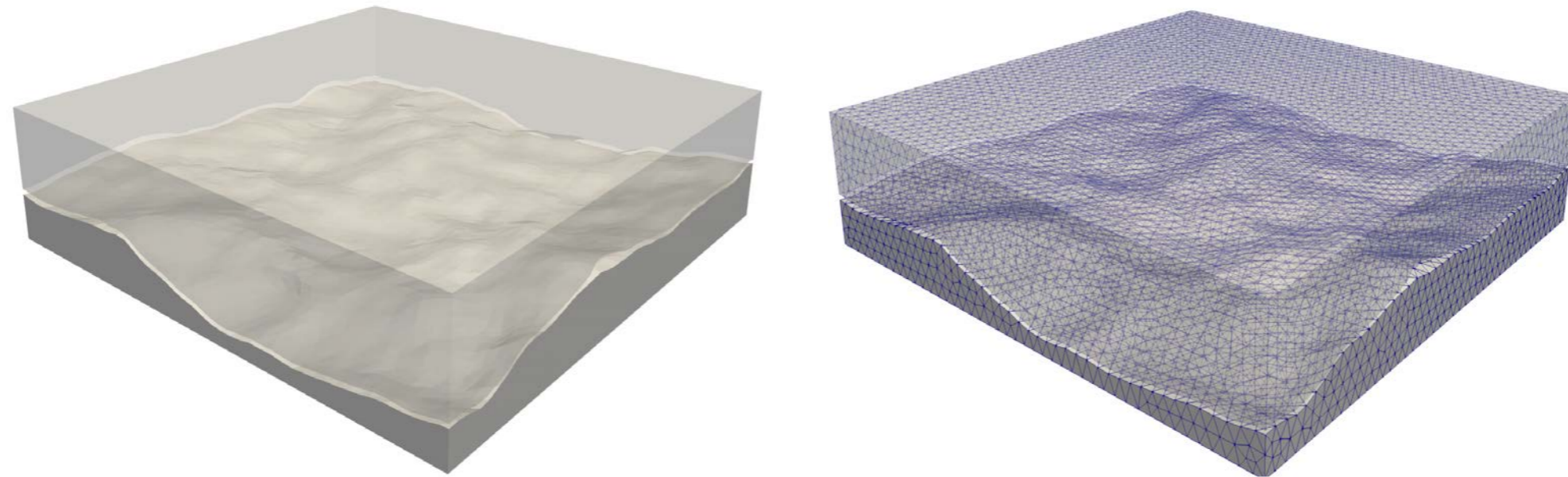


Proof of concept

Mesh-tying and FSI on idealised turbines (2)

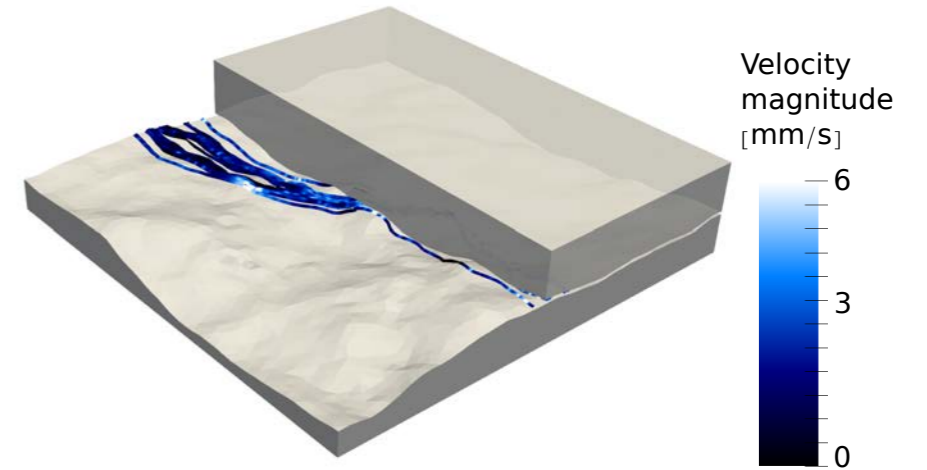


FSI in rough rock fractures (set-up)



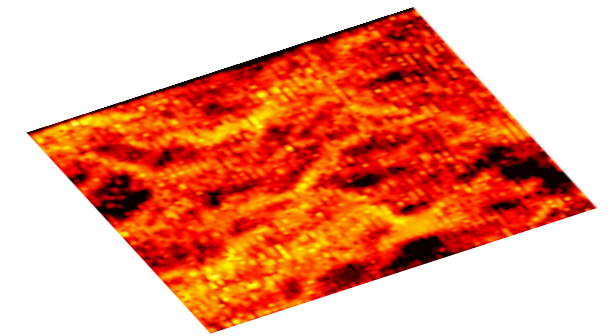
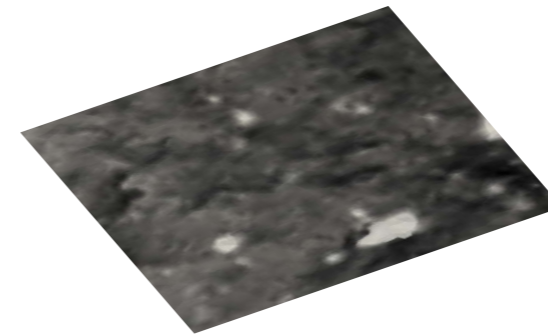
Literature: C. v. Planta and others. **Simulation of hydro-mechanically coupled processes in rough rock fractures using an immersed boundary method and variational transfer operators.** To be submitted.

FSI in rough rock fractures

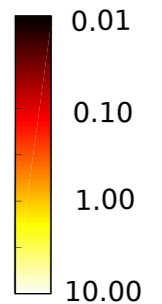
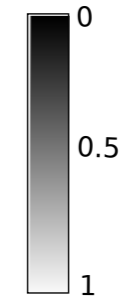
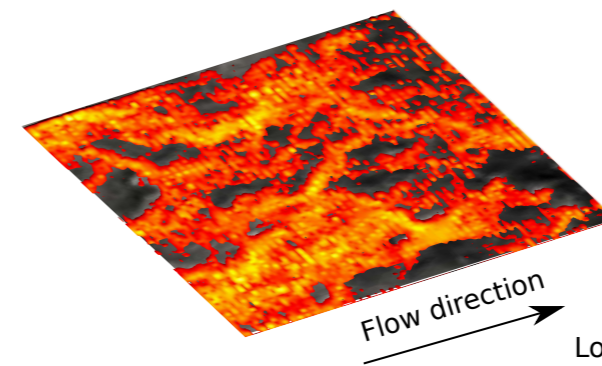


(a) Aperture distribution

(b) Fluid flow

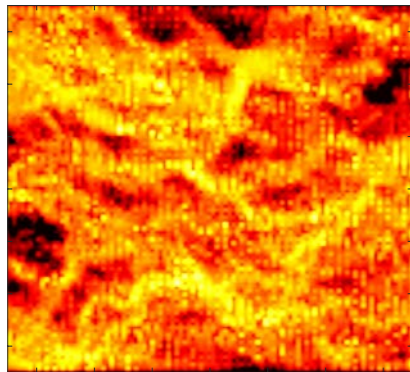
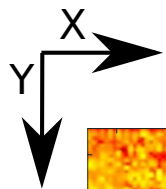


(c) Channeling flow in the heterogeneous aperture

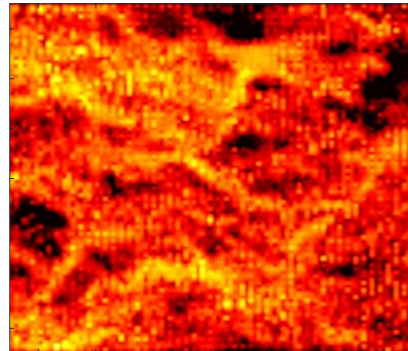


Local aperture [mm]

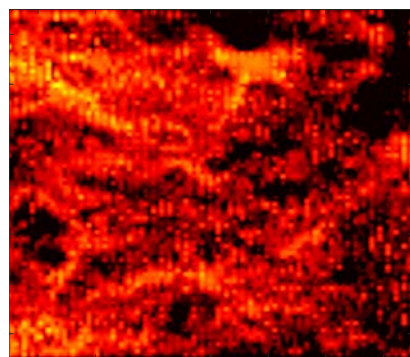
Local flow rate [mm³/s]



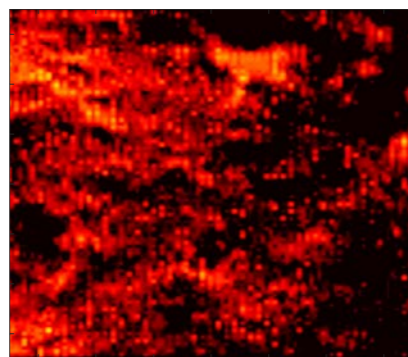
(a)



(b)

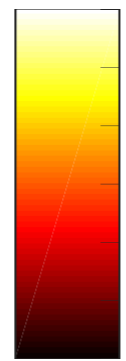


(c)



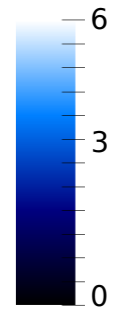
(d)

Flow rate [mm³/s]

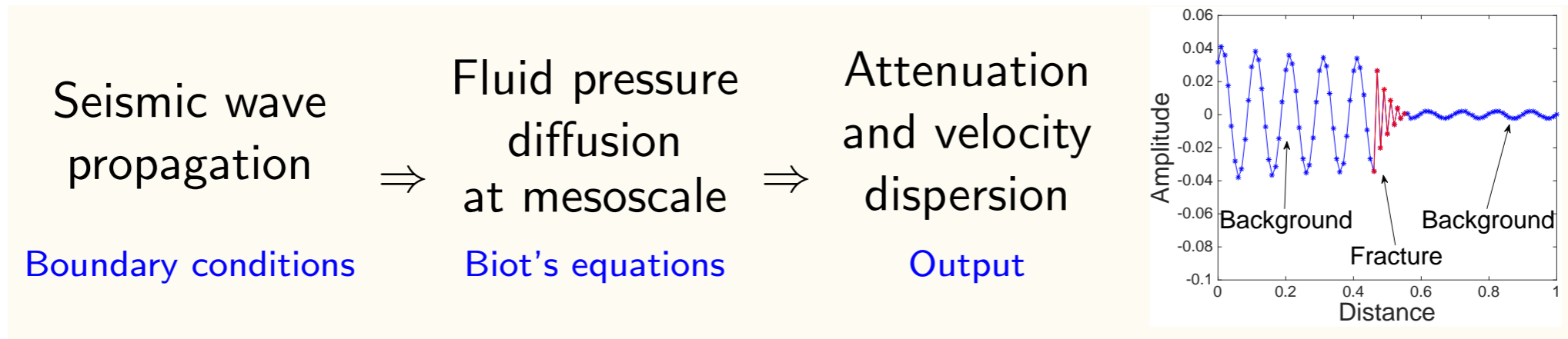


10.00
3.10
1.00
0.31
0.10
0.03
0.01

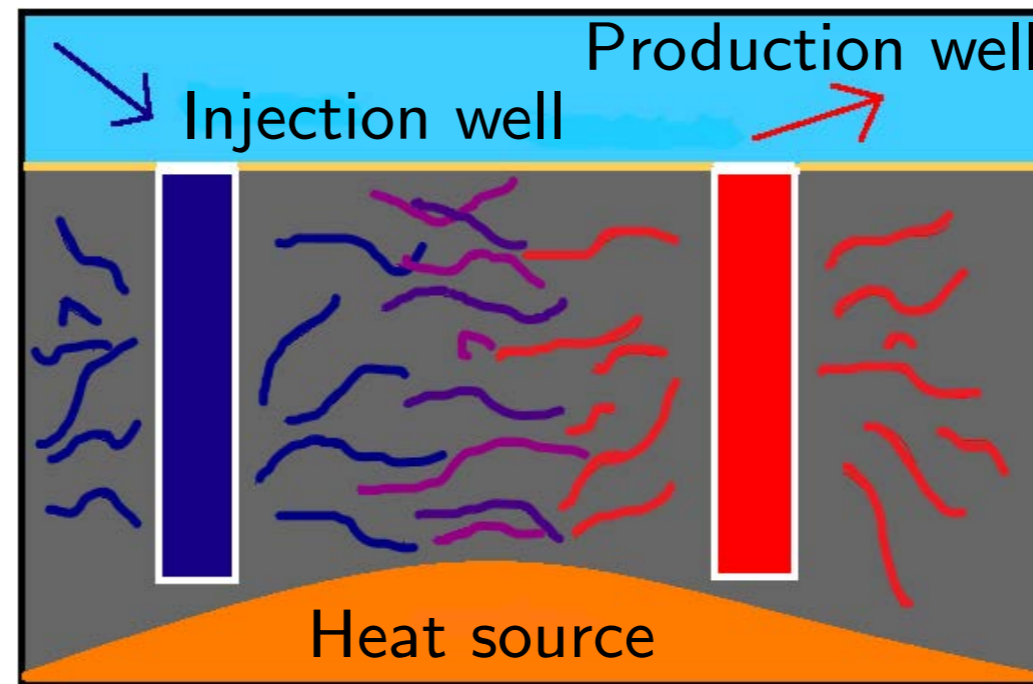
Velocity magnitude [mm/s]



Fluid-saturated fractured porous rocks

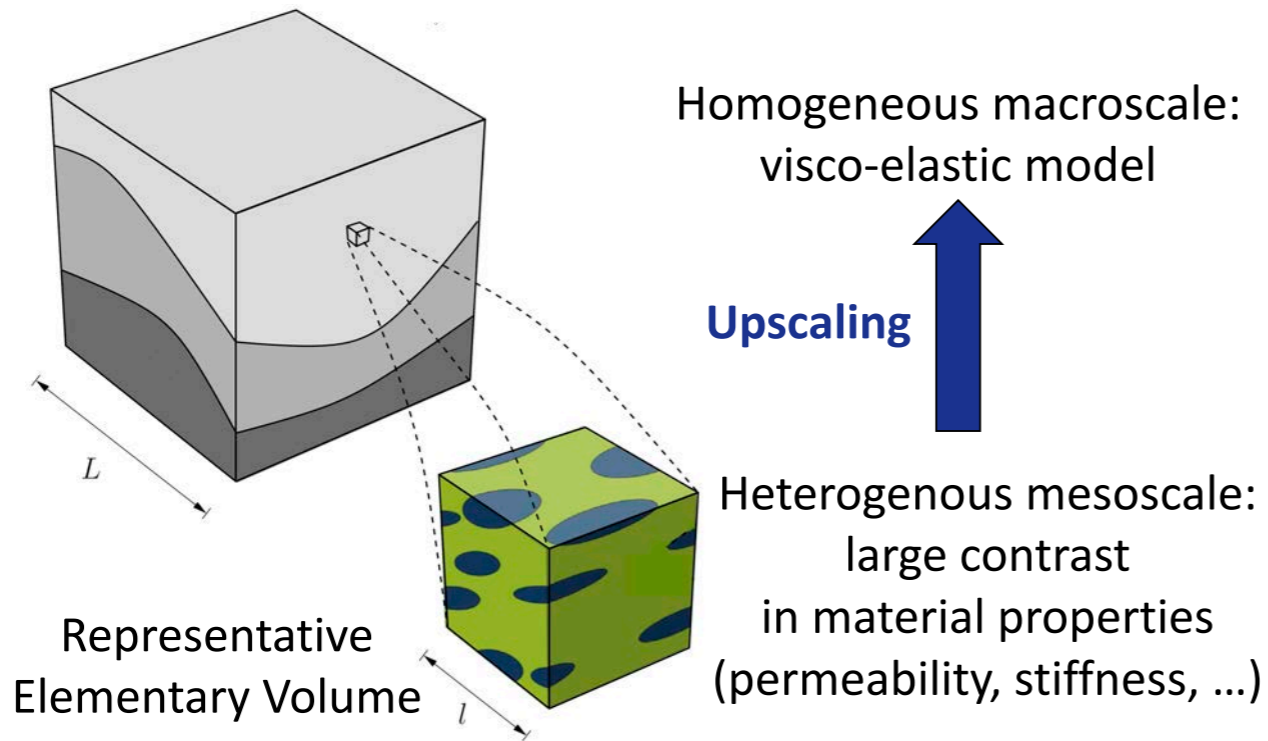


geometric + mechanical + hydraulic properties of fracture networks in rock formations



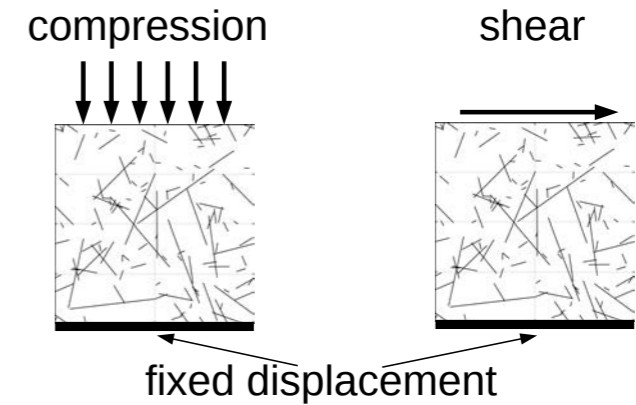
www.energybc.ca

Fractures as fluid pathways



Adapted from Jänicke et al. (2015)

Numerical Upscaling Experiments



periodic: displacement, stress, fluid pressure and the flux of the pore fluid

↓
to evaluate frequency-dependent

- attenuation
 - velocity dispersion
- } Y_ω

Hybrid-dimensional model: lower dimensional fractures

- simplified physics
- simple geometries

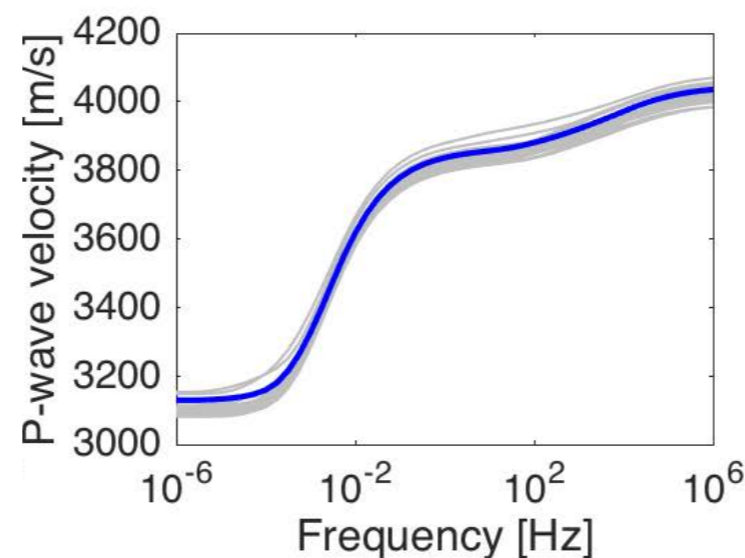
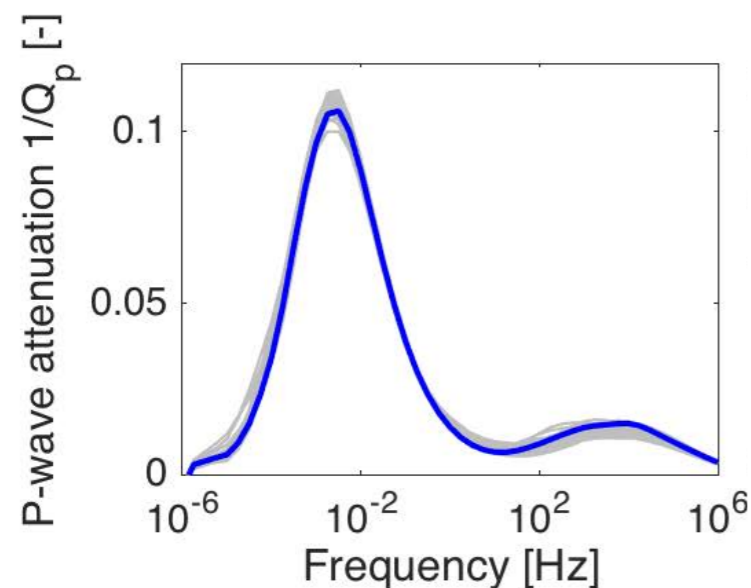
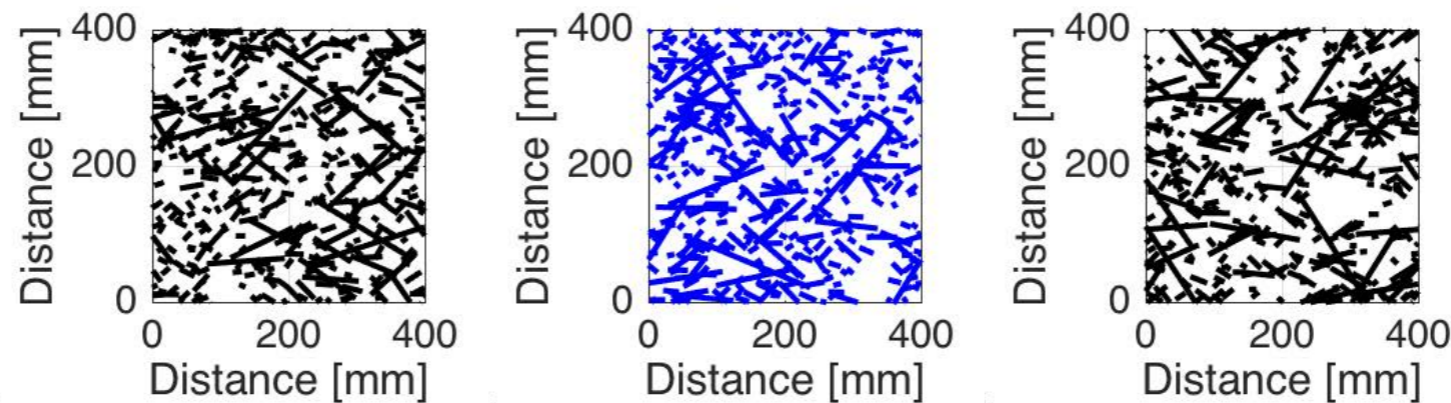
Biot's equations: "thick" fractures ✓

- complete coupled physics
- complex fracture geometries

Fracture distribution in a Representative Elementary Volume

- deterministic information not available or insufficient ✗
- statistical properties ✓

Monte Carlo method: N samples to estimate $\mathbb{E}(Y_\omega)$



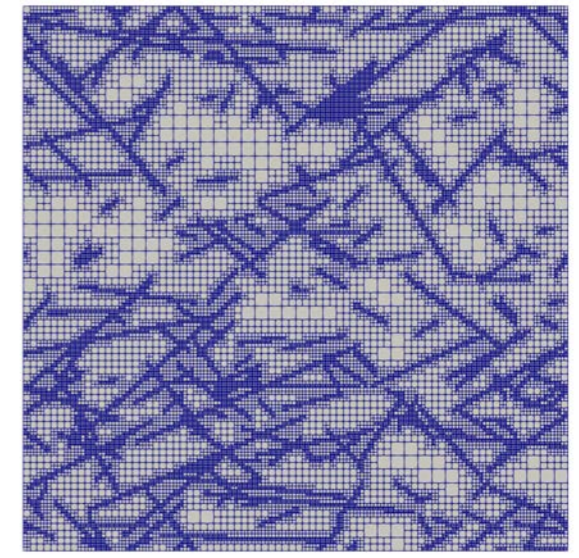
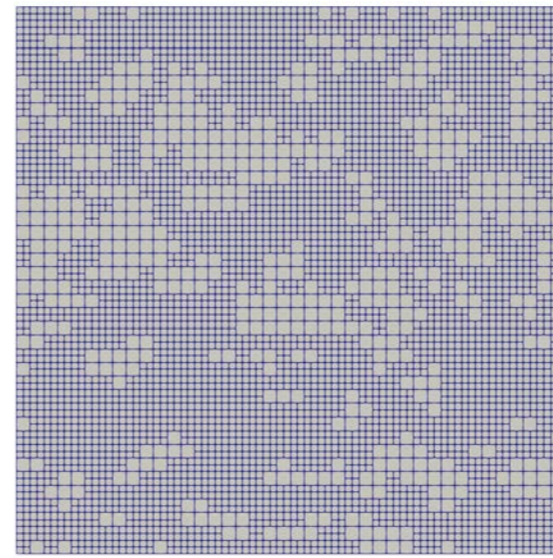
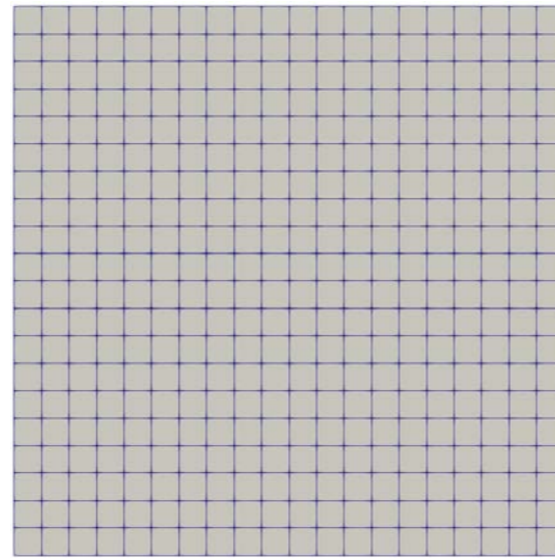
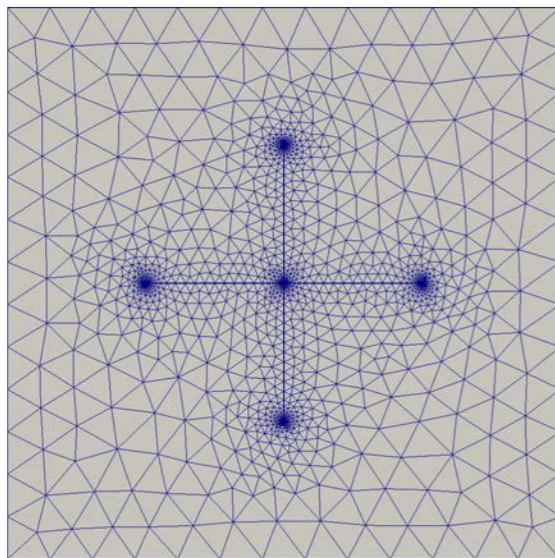
Monte Carlo
approximation

$$\frac{1}{N} \sum_{n=1}^N Y_\omega^n$$

Multi-scale problem

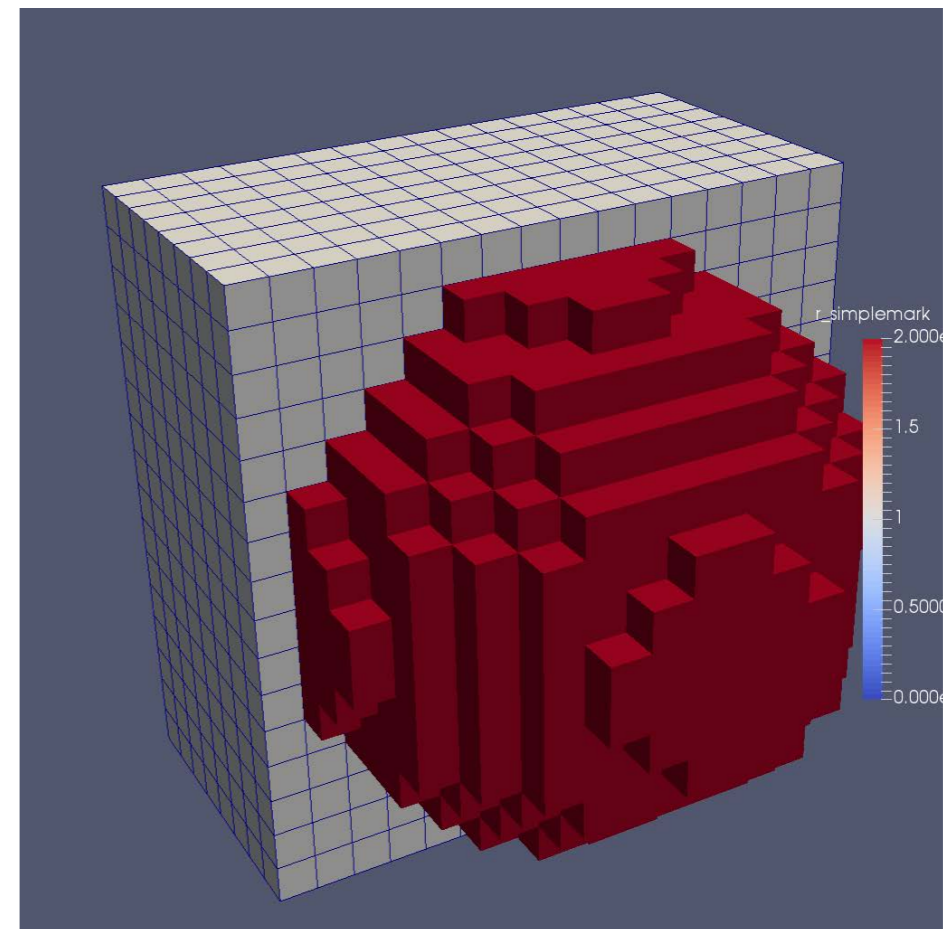
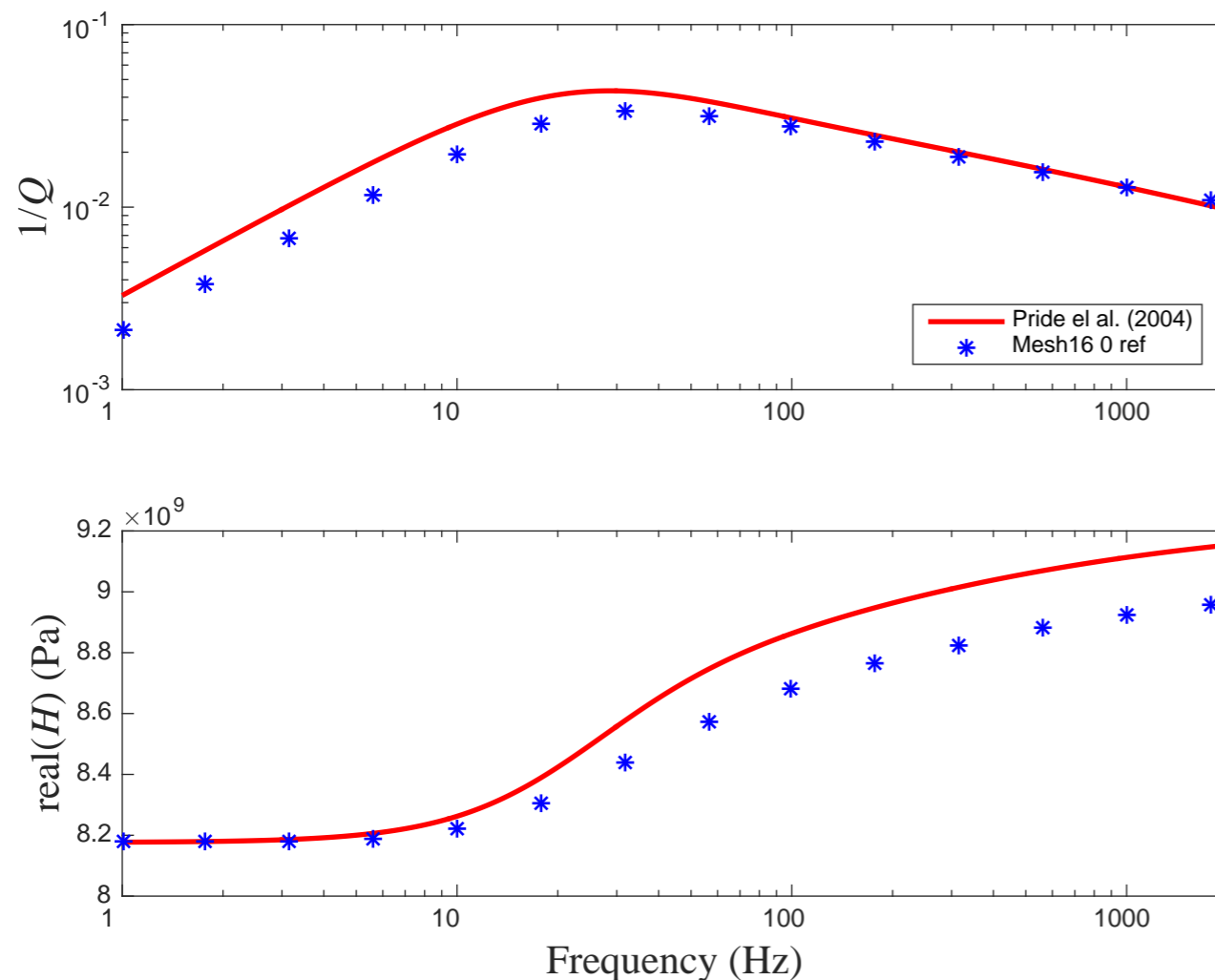
- Fracture thickness 0.1% of domain size
- 50 to 200 fractures
- **Conforming mesh generation**
 - Hands-on
 - Time consuming

- **Adaptive mesh-refinement**
 - Automatic
 - Complex fracture networks



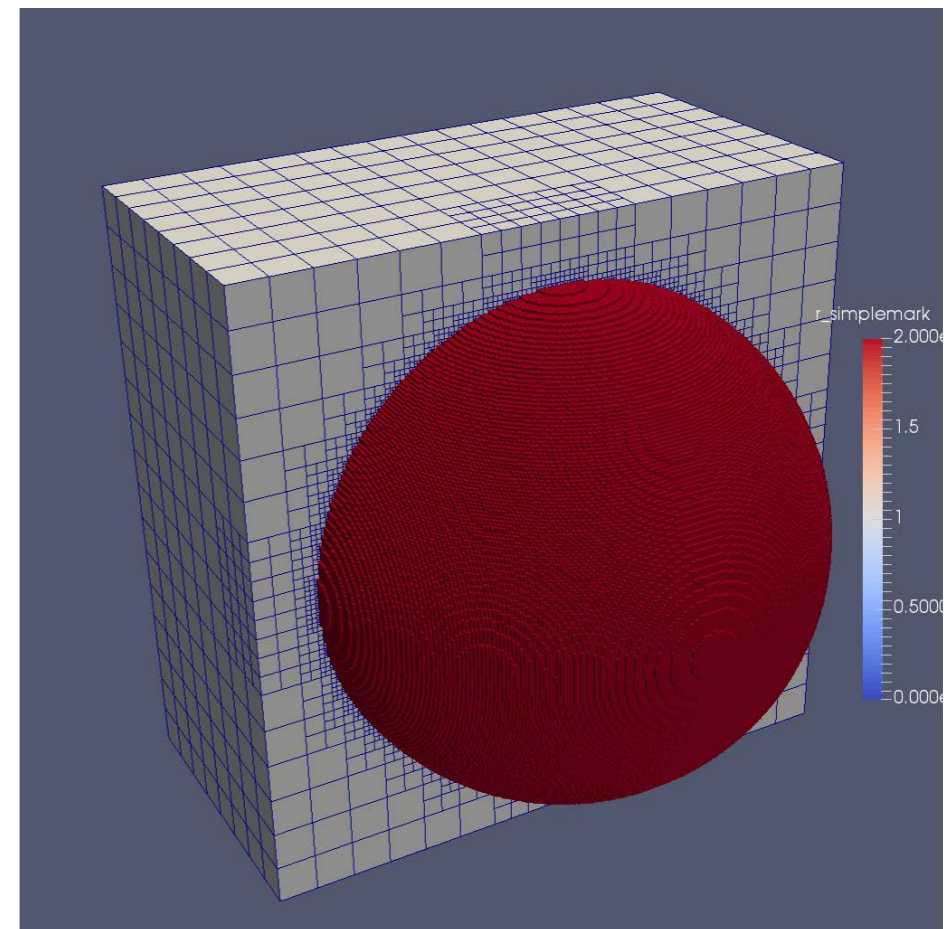
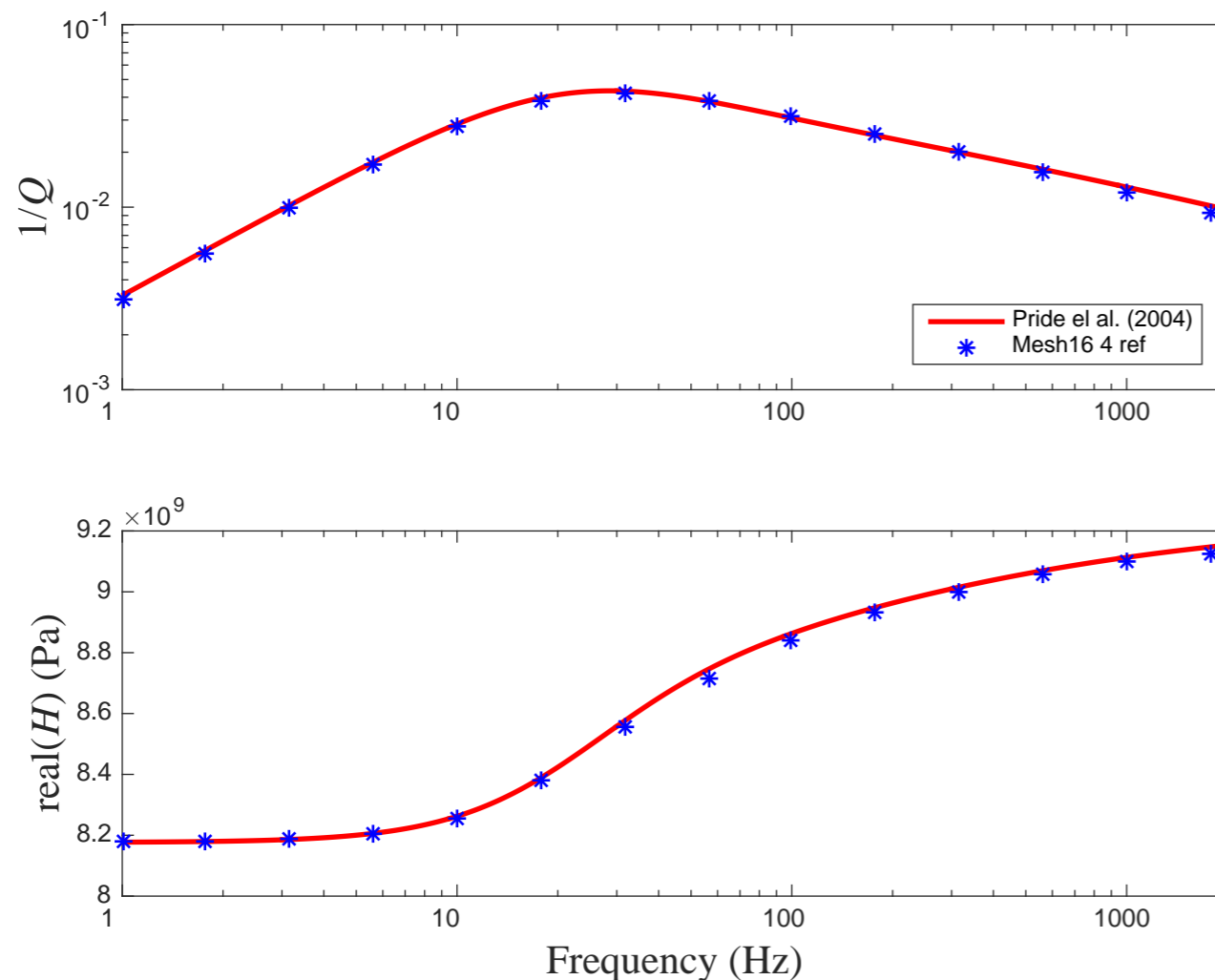
Convergence to analytical solution

- No difference between adaptive (**2.9 M nodes**) and uniform refinement (**135 M nodes**) for same minimum mesh size



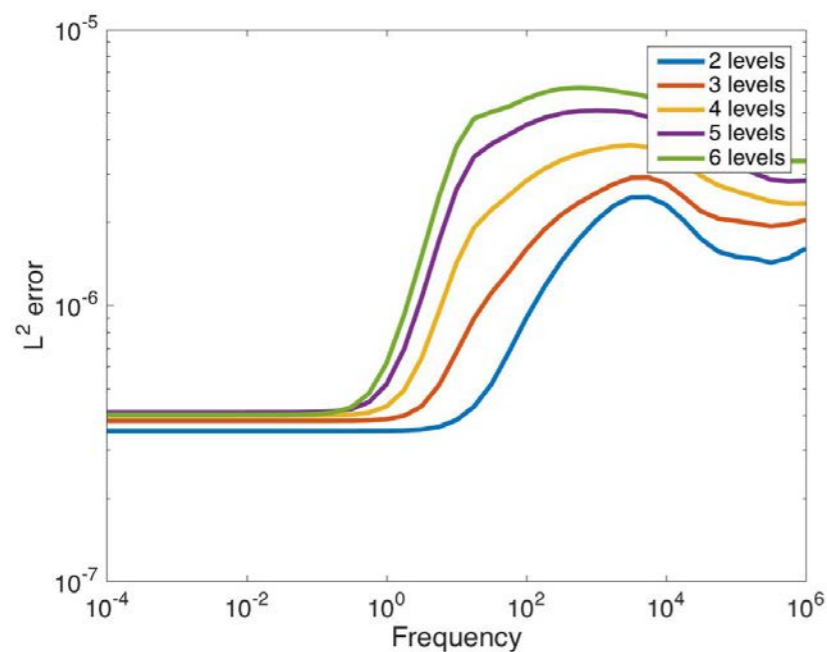
Convergence to analytical solution

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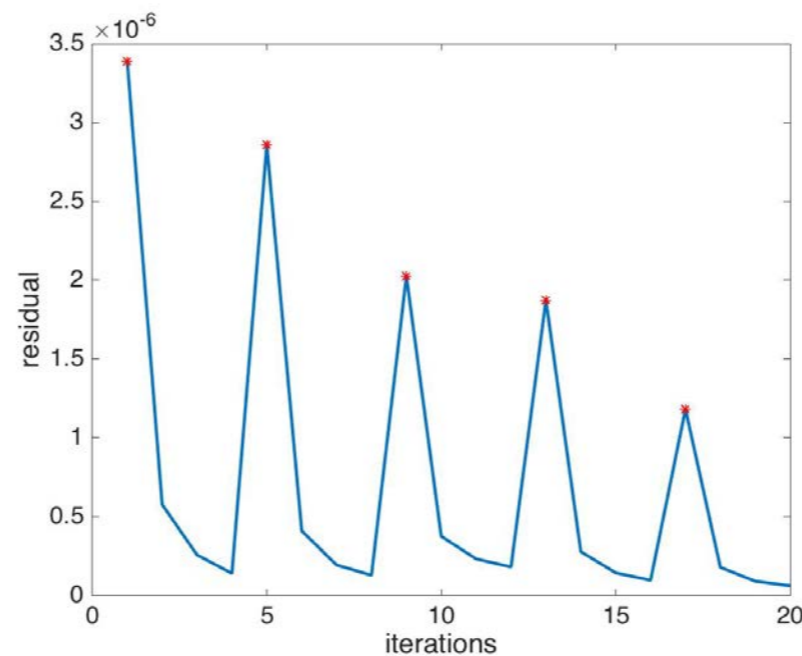


Sequence of solution exploiting refinement levels

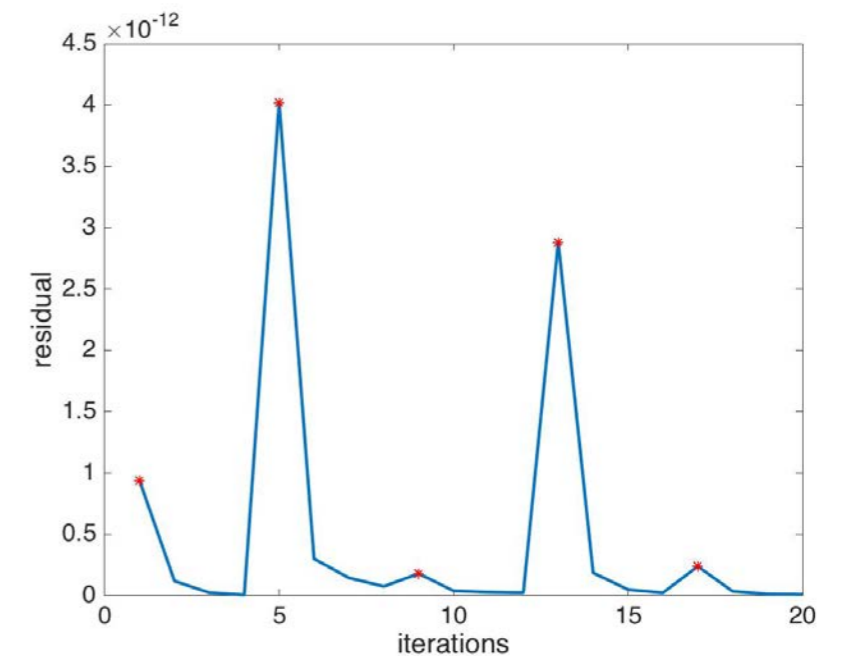
- Solution for coarsest space
- Coarse space solution is projected (**L²–projection**) onto the fine-space
- 3-post-smoothing steps of Block-Gauss-Seidel algorithm



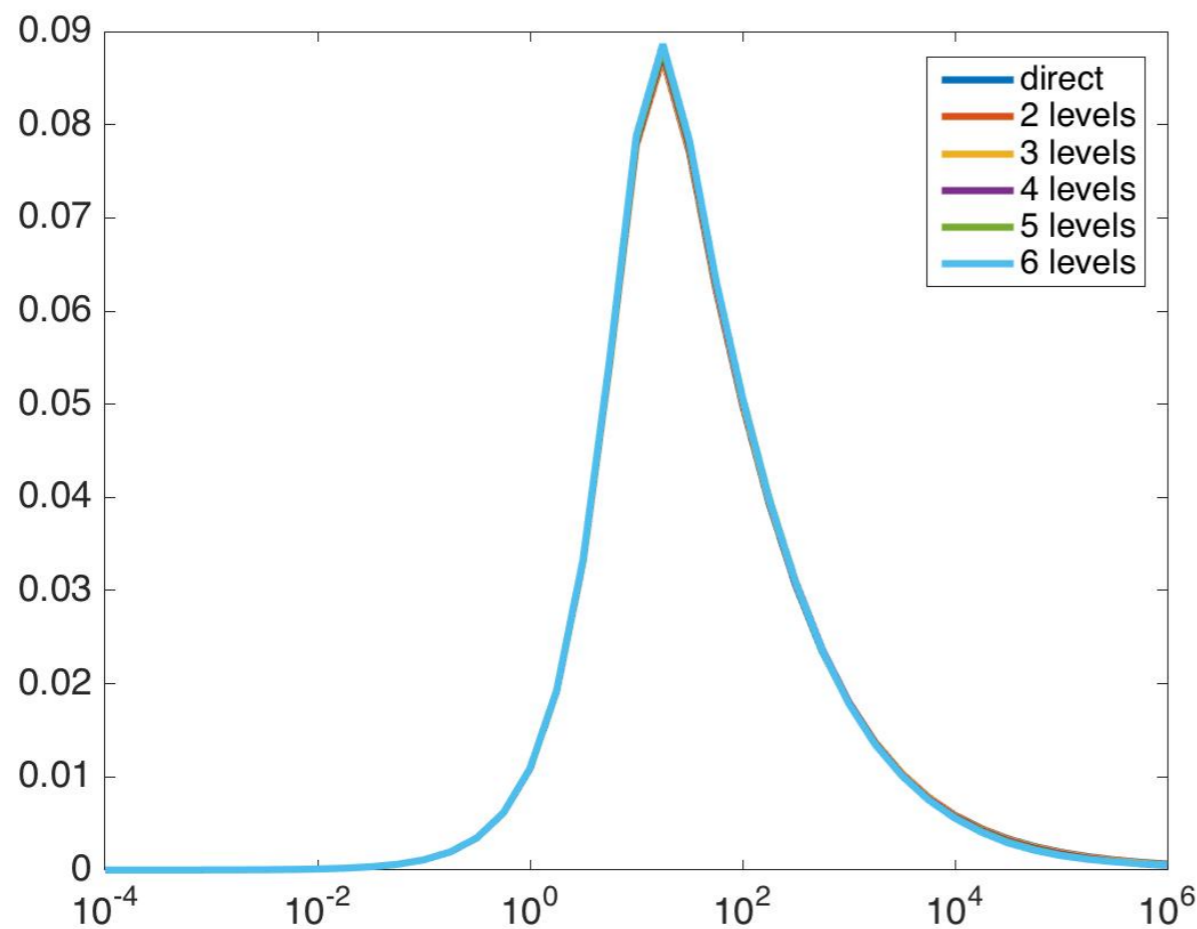
L²–error direct solver vs. MG



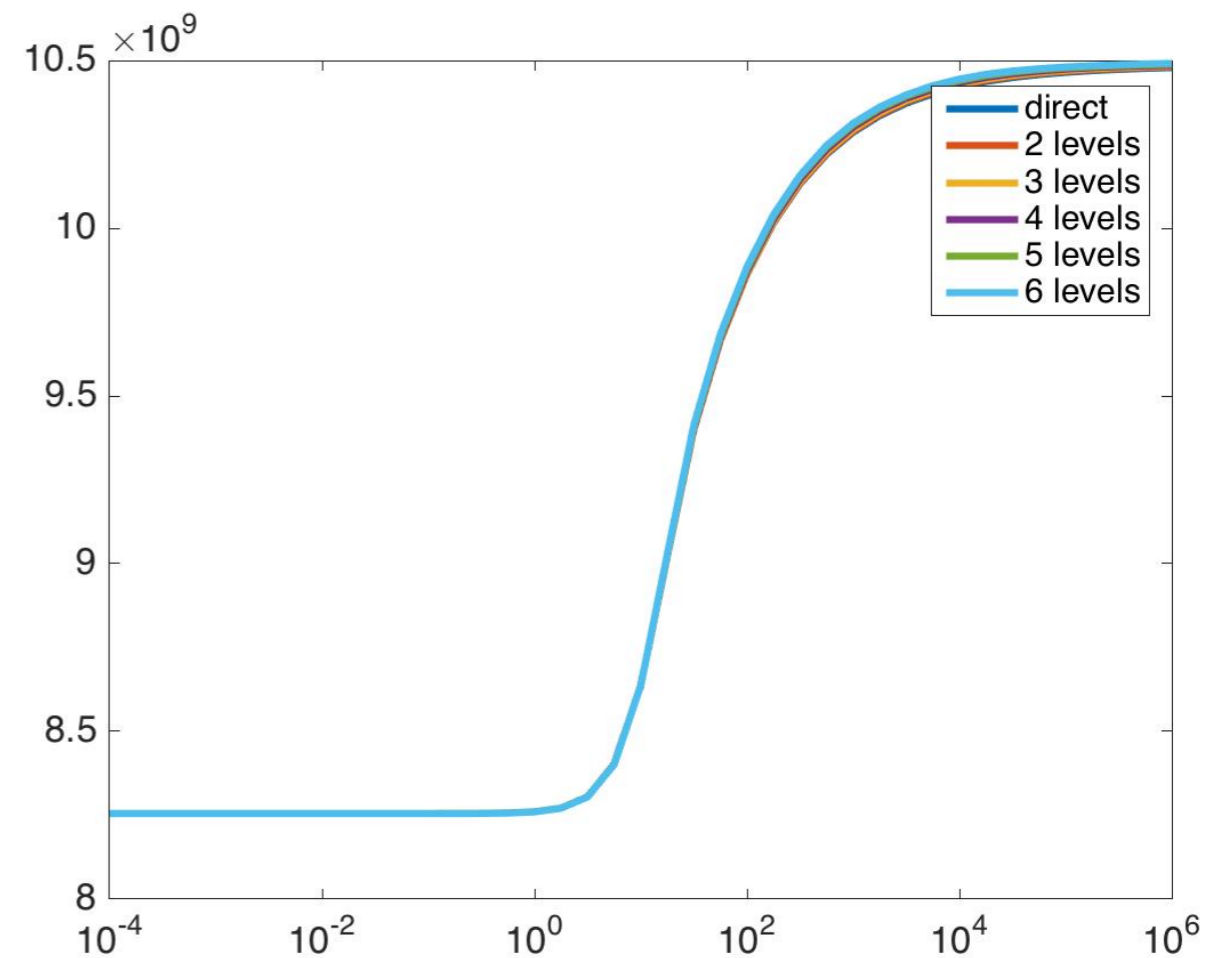
Residual–error history low freq — high freq



Adaptivity and low-cost MG strategy and **uniform and direct solver strategy** provide same P-wave attenuation and velocity dispersion curves



P-wave attenuation



Velocity dispersion

Utopia

bitbucket.org/zulianp/utopia

In collaboration with



CSCS

Centro Svizzero di Calcolo Scientifico
Swiss National Supercomputing Centre

ParMOONoLith

bitbucket.org/zulianp/par_moonolith

MFEM-MOONoLith

[github.com/mfem/mfem/tree/
moonolith-dev](https://github.com/mfem/mfem/tree/moonolith-dev)

In collaboration with



Presented today

- **Fictitious domain**/mortar element method for FSI
- Parallel **L²—projection** for volume and surface coupling
- **Multilevel** method for **stochastic fracture** networks, validated for sphere inclusion

Software

- **Moonolith + Utopia** (L2—projections)
- **Parrot** (Poro-elasticity for fractures)
- **MOOSE** (FEM)

Future work

- **Multigrid** method for realistic fracture networks and **Multilevel Monte Carlo**

Thank you for your attention

See you in Lugano at



Important Dates

Opening of minisymposia proposals	August 15, 2018
Minisymposia acceptance notification	November 15, 2018
Opening of abstract submissions	November 15, 2018
Deadline for abstract submissions	January, 15, 2019
Abstract acceptance notification	February 15, 2019

Articles related to this talk

A Parallel Approach to the Variational Transfer of Discrete Fields between Arbitrarily Distributed Unstructured Finite Element Meshes. Rolf Krause and Patrick Zulian. SIAM Journal on Scientific Computing, 2016.

An immersed boundary method based on the L2-projection approach. Maria Giuseppina Chiara Nestola, Barna Becsek, Hadi Zolfaghari, Patrick Zulian, Dominik Obrist, and Krause Rolf. Proceedings of the 24rd International Conference on Domain Decomposition Methods, 2018.

An immersed boundary method for fluid-structure interaction based on overlapping domain decomposition. Maria Giuseppina Chiara Nestola, Barna Becsek, Hadi Zolfaghari, Patrick Zulian, Dario Demarinis, Dominik Obrist, and Krause Rolf. Journal of computational physics (in review).