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A Fictitious Domain Method for FSI Simulations

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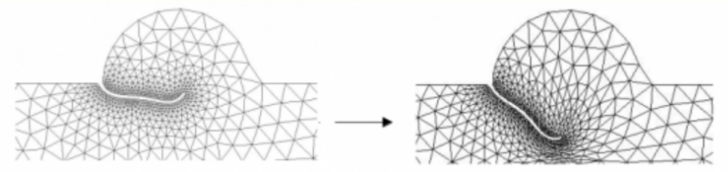
a Development of a software for simulating the interaction between a fluid and a solid structure based on the embedded boundary method

b Efficient handling of the data transfer between the fluid and the solid mesh

- **Several approaches** have been developed to reproduce the **interaction** between a **fluid** and a **solid structure**:

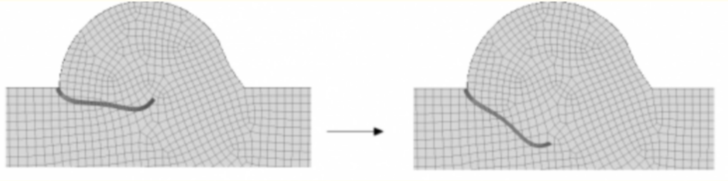
Boundary-fitted method



Guarantees accurate results at the interface between the solid structure and the fluid flow.
Scenarios with **large displacements** → **distorted fluid grid** may affect the numerical stability of the problem and the accuracy of the solution.

- **Several approaches** have been developed to reproduce the **interaction** between a **fluid** and a **solid structure**:

Embedded Boundary method



Designed to embed the solid phase within the fluid phase, enabling the calculation of the FSI effect on a stationary fluid grid which can be analysed in a purely Eulerian fashion.

Solving the FSI problem implies the necessity to couple a fluid and a structure problem.

Requirement

1. **transfer of data** between **different**
   - **non matching meshes**
   - **non conforming approximation spaces**
2. numerical simulations of **complex** and **large scale** problems
3. use of supercomputers: **meshes aritrarialy distributed among processors**

The way the transfer operators are constructed affects **convergence**, **accuracy** and **efficiency**

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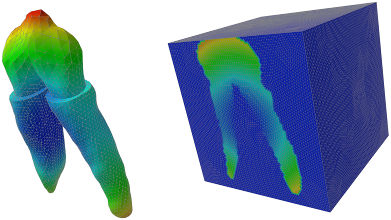
L^2-**projection MOONoLith**: library developed at ICS (http://moonolith.inf.usi.ch)

Krause, Rolf, and Patrick Zulian. "A Parallel Approach to the Variational Transfer of Discrete Fields between Arbitrarily Distributed Unstructured Finite Element Meshes." SIAM Journal on Scientific Computing 38.3 (2016): C307-C333.)

MOONoLith

(http://moonolith.inf.usi.ch)

Transfer the data from a **source** space to a **target** space



Source and Target Mesh

$\Omega_v, \Omega_w \subset \mathbb{R}^d \rightarrow$ bounded domains approximated by Ω_v^h and Ω_w^h,

$\mathcal{T}_v^h$ and $\mathcal{T}_w^h \rightarrow$ associated meshes,

$V_h = V_h(\mathcal{T}_v^h)$ and $W_h = W_h(\mathcal{T}_w^h) \rightarrow$ associated spaces.

Description of the L^2 projection

For the definition of the projection operator, one needs to define a **suitable discrete space of Lagrange multipliers** M_h.
Set M_h as a **discrete space** based on **the same space as the target space**.

$$\int_{I_h} (v_h - P(v_h))\mu_h \, d\mathbf{x} = \int_{I_h} (v_h - w_h)\mu_h \, d\mathbf{x} = 0 \quad \forall \mu_h \in M_h. \tag{1}$$

$$\Downarrow$$

$$\sum_{i \in J_v} v_i \int_{I_h} \phi_i \psi_k \, d\mathbf{x} = \sum_{j \in J_w} w_j \int_{I_h} \theta_j \psi_k \, d\mathbf{x} \qquad \text{for } k \in J_\mu. \tag{2}$$

$$\Downarrow$$

$$\mathbf{w} = \mathbf{D}^{-1}\mathbf{B}\mathbf{v} = \mathbf{T}\mathbf{v}.$$

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$$\Downarrow$$

pseudo L^2 projection

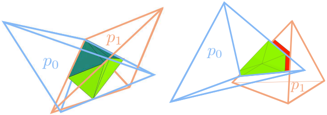
$$(\psi_k, \theta_j)_{L^2(I_h)} = \delta_{j,k} (\theta_j, \mathbf{1})_{L^2(I_h)} \quad \forall j, k \in J_w. \tag{5}$$

$$\Downarrow$$

$\mathbf{D} \Rightarrow$ diagonal matrix

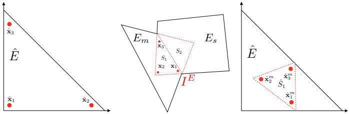
Description of the L^2 projection

1. Transfer data, $\mathbf{v}$, from E_m to data, $\mathbf{w}$, on E_s requires finding mesh intersections for quadrature.



2. Intersection-Detection: parallel tree-search algorithm. Outcome: element pairs (associated with processes).

3. Generate the quadrature points for integrating in the intersection region I^E.

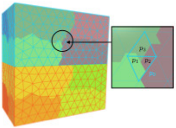


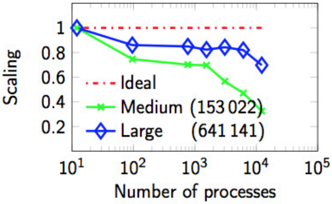
Description of the L^2 projection

4. Compute the local element-wise contributions for the operators $\mathbf{B}$ and $\mathbf{D}$. Assemble one matrix $\mathbf{T}$ containing all the different projection matrices $\mathbf{T}_{m,s}$ for every pair of intersecting meshes:

$$\mathbf{T} = \begin{bmatrix} \mathbf{T}_{1,1} & \mathbf{T}_{1,2} & \cdots & \mathbf{T}_{1,n} \\ \vdots & & \ddots & \vdots \\ \mathbf{T}_{n,1} & \mathbf{T}_{n,2} & \cdots & \mathbf{T}_{n,n} \end{bmatrix}$$

Weak Scaling Tests





Find $(\mathbf{u}_f, p_f; \boldsymbol{\eta}_s, p_s; \boldsymbol{\lambda}) \subset (V_f \times Q_f \times V_s \times Q_s \times L)$ such that for every $(\mathbf{v}_f, q_f; \mathbf{v}_s, q_s; \boldsymbol{\mu}) \subset (V_f \times Q_f \times V_s \times Q_s \times L)$

$$\int_{\Omega_f} \rho_f \frac{\partial \mathbf{u}_f}{\partial t} \cdot \mathbf{v}_f dV + \int_{\Omega_f} \rho_f [(\mathbf{u}_f \cdot \nabla)\mathbf{u}_f] \cdot \mathbf{v}_f dV + \int_{\Omega_f} \sigma(\mathbf{u}_f, p_f) : \nabla \mathbf{v}_f dV - \int_{\mathcal{I}} \boldsymbol{\lambda} \cdot \mathbf{u}_f dV = 0$$

$$\int_{\Omega_f} q_f \nabla \cdot \mathbf{u}_f dV = 0$$

$$\int_{\mathcal{I}} \boldsymbol{\mu} \cdot \left(\frac{\partial \boldsymbol{\eta}_s}{\partial t} - \mathbf{u}_f \right) dV = 0$$

$$\int_{\widehat{\Omega}_s} \widehat{\rho}_s \frac{\partial^2 \widehat{\boldsymbol{\eta}}_s}{\partial t^2} \cdot \widehat{\mathbf{v}}_s + \int_{\widehat{\Omega}_s} \widehat{\mathbf{P}}(\widehat{\mathbf{F}}) : \nabla \widehat{\mathbf{v}}_s dV + \int_{\mathcal{I}} \boldsymbol{\lambda} \cdot \mathbf{v}_s dV = 0$$

$\widehat{\Omega}_s$: solid domain, Ω_f : fluid domain $\mathcal{I} := \widehat{\Omega}_s \cap \Omega_f$.

"A fictitious domain/mortar element method for fluid-structure interaction." International Journal for Numerical Methods in Fluids 35.7 (2001): 743-761.

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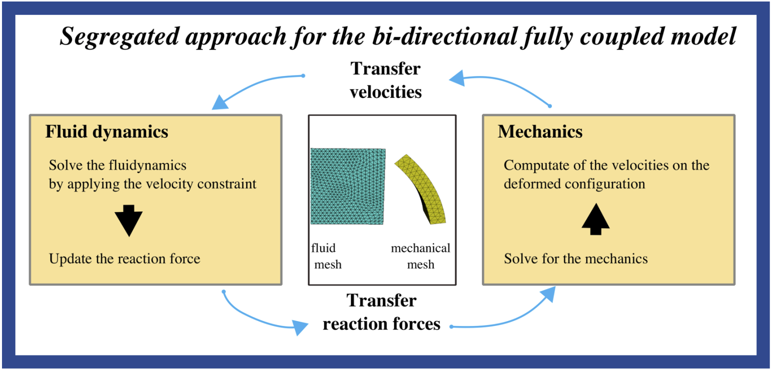
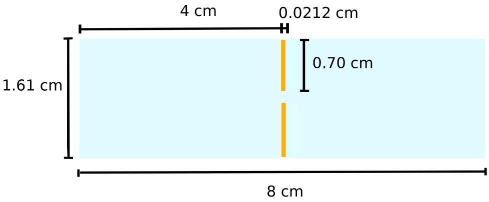
Segregated Approach

1. decoupled variables
2. iterations between subproblems
3. fixed point iteration

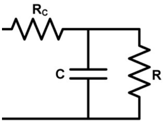


2D Benchmark



Boundary Conditions

1. Inlet: Parabolic Profile $v_{fluid}(t) = 5y(y - 1.61)[\sin(2\pi t) + 1.1]$
2. Outlet: Windkessel Model



$$Q(t) = C\frac{dP(t)}{dt} + \frac{P(t)}{R} \quad P_{outlet}(t) = Q(t) \cdot R_c + P(t)$$

3. No slip boundary conditions on the top and on the bottom

Gil, Antonio J., et al. J. Computational Physics 229 (2010): 8613-8641.

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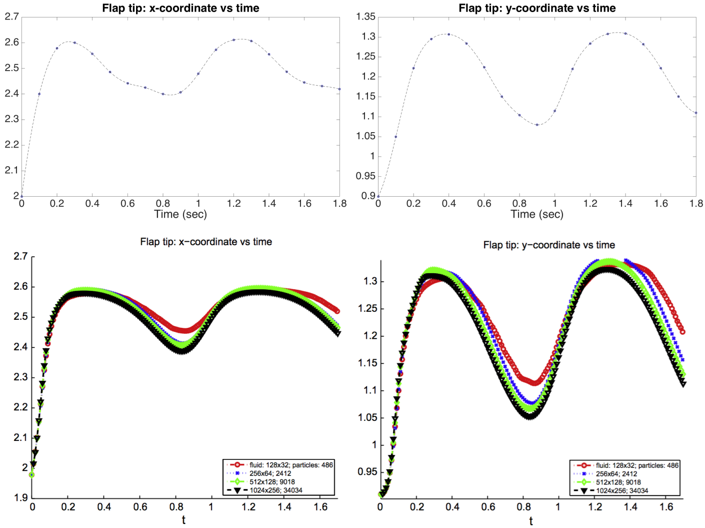
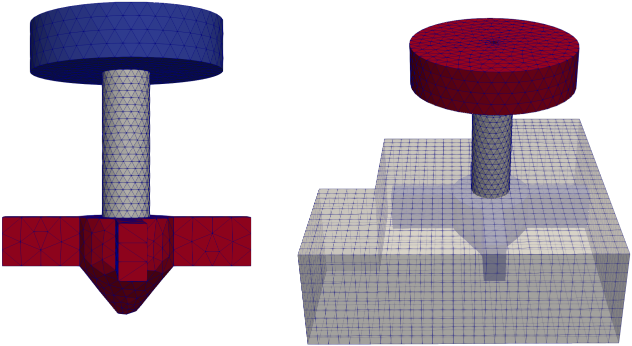
1. Neo-Hookean constitutive model: Beams with different stiffness
2. Newtonian fluid



2D Benchmark

Towards 3D Simulations of Turbines

1. Linear constitutive model for the turbine blades
2. Newtonian fluid, Reynolds Number=2000
3. Non Conforming Meshes (Fluid and Solid Grid)



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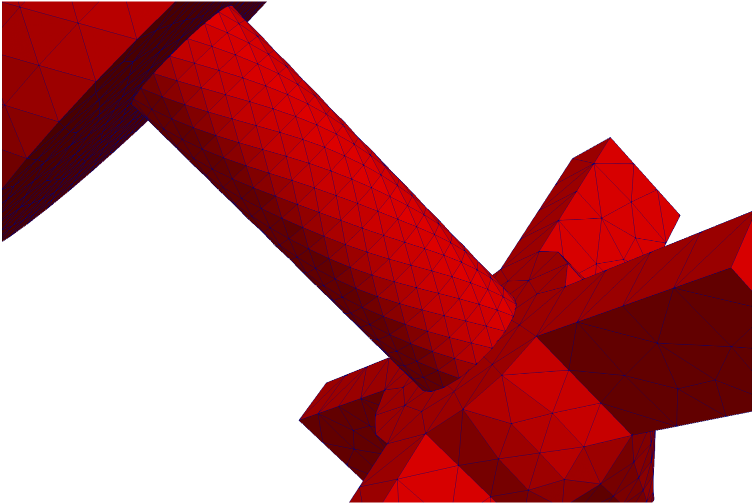
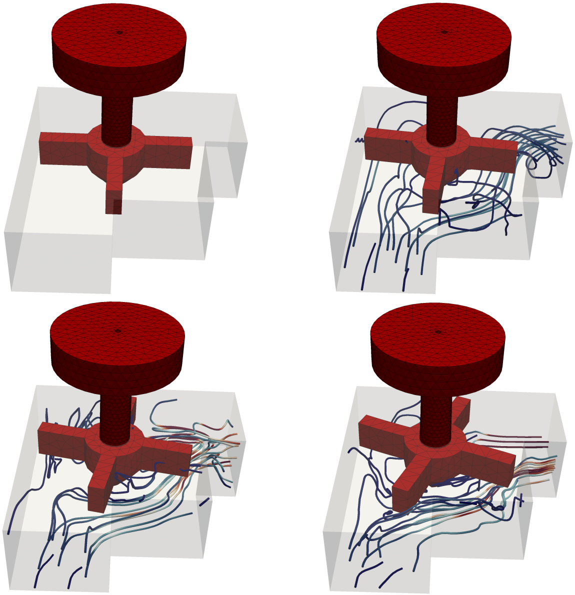
1. Non Conforming Meshes



Towards 3D Simulations of Turbines



Towards 3D Simulations of Turbines

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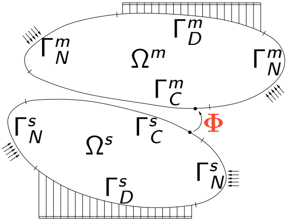
Contact in linear elasticity:problem setting

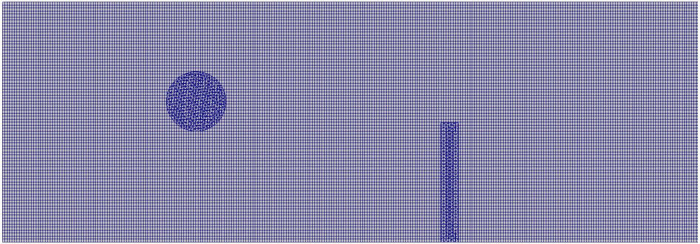


- $\Omega = \Omega^m \cup \Omega^s \subset \mathbb{R}^3$ two bodies
  $\Gamma_D = \Gamma_D^m \cup \Gamma_D^s$ Dirichlet boundary
  $\Gamma_N = \Gamma_N^m \cup \Gamma_N^s$ Neumann boundary
  $\Gamma_C = \Gamma_C^m \cup \Gamma_C^s$ possible contact boundary
- Find displacement $\boldsymbol{u} = (\boldsymbol{u}^m, \boldsymbol{u}^s)$ s. t.

$$\begin{aligned} -\sigma_{ij}(\boldsymbol{u})_{,j} &= f_i && \text{in } \Omega, \\ \boldsymbol{u} &= \mathbf{0} && \text{on } \Gamma_D, \\ \sigma_{ij}(\boldsymbol{u})n_j &= \bar{t}_i && \text{on } \Gamma_N. \end{aligned}$$

- $\boldsymbol{\Phi} : \Gamma_C^s \to \Gamma_C^m$: bijective "contact mapping", $\boldsymbol{n}^\Phi$: induced normal field, d: initial gap
  $[u] := (\boldsymbol{u}^s - \boldsymbol{u}^m \circ \boldsymbol{\Phi}) \cdot \boldsymbol{n}^\Phi$: jump in $\boldsymbol{n}^\Phi$-direction
- Enforce contact conditions on Γ_C:

$$\begin{aligned} \boldsymbol{\sigma}_t(\boldsymbol{u}^m) &= \boldsymbol{\sigma}_t(\boldsymbol{u}^s) = \mathbf{0} && \text{no friction} \\ \sigma_n(\boldsymbol{u}^m \circ \boldsymbol{\Phi}) &= \sigma_n(\boldsymbol{u}^s) \leq 0 && \text{force balance} \\ ([u] - d)\sigma_n(\boldsymbol{u}^s) &= 0 && \text{complementarity condition} \\ [u] &\leq d && \text{no penetration } (\textbf{linearized}) \end{aligned}$$



Contact in linear elasticity:results

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Simulate the interaction between the fluid and the solid

Conclusions

Simulate the interaction between the fluid and the solid

Solve contact problem

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Solve contact problem

Efficient and flexible way to treat the coupling on the volume on the interface by means of the $L^2 -$ *projection* approach

The use of the $L^2 -$ *projection* approach (MOONoLith library) allows for coupling arbitrary "in-house" solver codes based on different kind of spaces discretizations (Finite Element, Finite Volume, Finite differences), (i.e AV-Flow, (Artrog Center, Bern))

Simulate the interaction between the fluid and the solid

Solve contact problem

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Future Applications: flow in fractured networks,friction between rocks

Thank you for your attention