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Institute of Computational Science ICS

Efficient Finite Element Simulations for Fracture Networks

Task 3.2: Computational Energy Innovation

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Zurich, September 14-15, 2017

1. Development of a FE software to study
   - seismic attenuation
   - modulus dispersion

   due to fluid pressure diffusion in fractured rocks

2. Efficient for
   - stochastic fracture networks

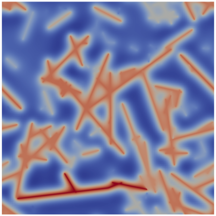
Model

- Biot's quasi-static equations
- fractured media (jumping parameters)
- time-frequency domain
  - $\mathbf{u}$ and p are complex variables

$$\begin{cases} -\nabla \cdot (2\mu\boldsymbol{\varepsilon} + \lambda \mathrm{tr}(\boldsymbol{\varepsilon})\mathbf{I} - \alpha p \mathbf{I}) &= 0 \\ i\omega\alpha\nabla \cdot \mathbf{u} + i\omega \frac{p}{M} + \nabla \cdot \left(-\frac{k}{\eta}\nabla p\right) &= 0 \end{cases}$$

Computational challenges

- mesh generation
- efficient solution methods for complex FE
  - two different discretization approaches



$$
\begin{vmatrix} A & -B^T \\ -iB & -iM - \frac{1}{\omega}C \end{vmatrix} \begin{vmatrix} \mathbf{u} \\ \mathbf{p} \end{vmatrix} = \begin{vmatrix} \mathbf{f} \\ \mathbf{0} \end{vmatrix}
$$

Complex FE

- 4 variables in 3D
- `complex<double>` type (two doubles for each entry)
- not well-conditioned
- better for factorization (direct solvers)
- Generalized Saddle-point problem
  - no energy
  - not symmetric $\Rightarrow$ no Lagrangian
  - requires ad-hoc solution methods
- e.g. Comsol

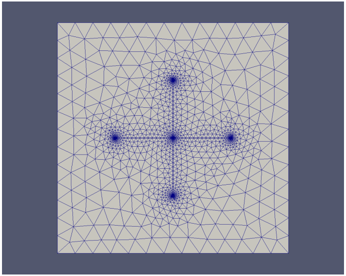
$$\begin{vmatrix} A & 0 & -B^T & 0 \\ 0 & A & 0 & -B^T \\ 0 & B & -\frac{1}{\omega}C & -M \\ B & 0 & M & -\frac{1}{\omega}C \end{vmatrix} \begin{vmatrix} \mathbf{u}_\mathrm{r} \\ \mathbf{u}_\mathrm{i} \\ \mathbf{p}_r \\ \mathbf{p}_i \end{vmatrix} = \begin{vmatrix} \mathbf{f}_\mathrm{r} \\ \mathbf{f}_\mathrm{i} \\ \mathbf{0} \\ \mathbf{0} \end{vmatrix}$$

Real FE

- 8 variables
- `double` type (one double per entry)
- better condition number
- better for iterative solvers
- Generalized Saddle-point problem
  - no energy
  - not symmetric $\Rightarrow$ no Lagrangian
  - requires ad-hoc solution methods

Multiscale problem:

- fracture thicknesses $\simeq 10^{-3}$ of domain size
- fractures need to be resolved to set correct parameters



- Meshing is one of the bottlenecks of the problem ✗
  - elements follow the geometry
  - hands-on
  - time consuming
  - may fail

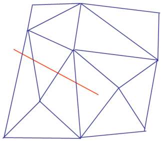
$\Rightarrow$ unfeasible for realistic networks

FE theory does not require fracture resolution

- Consider we have to compute

$$C_{ij} = \int_{T_h} \frac{k}{\eta} \nabla \phi_j \cdot \nabla \phi_i \, dx$$

need area (or volume) of element covered by fracture(s)



- homogenized approach ✗
  - mesh can be too coarse for considered application
  - composite FE may improve it (Hackbusch & Sauter, 1997)
- expensive ✗
  - complicated geometric search in order to find the area, or
  - complicated quadrature rules

  $\Rightarrow$ "enough" quadrature points have to be in the fracture

Adaptive mesh refinement (AMR)

- elements do not follow the geometry but refined close to the interfaces
- more elements where error is larger
- automatized
- cannot fail
- readily allows for random fracture distributions (stochastic simulations)
- coarser levels can be used for multigrid and multilevel Montecarlo simulations

Mesh is generated once and then used for several frequencies

2D (5 parameters)

- center point (x,z)
- thickness and length
- dip around y-axis

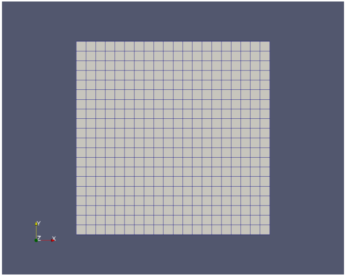
3D (8 parameters)

- center point (x,y,z)
- thickness, length and width
- dip around y-axis and x-axis

Parameters can be drawn from any distribution (e.g. de Dreuzy, Normal)

Example with 6 fractures

- mesh adapted outside the fracture
- material properties non-constant on each element
- continuity imposed at hanging nodes
- mesh adapted periodically



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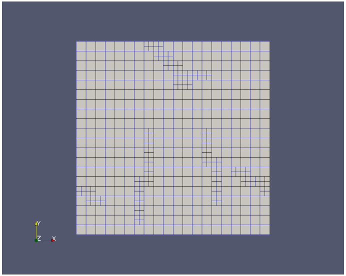

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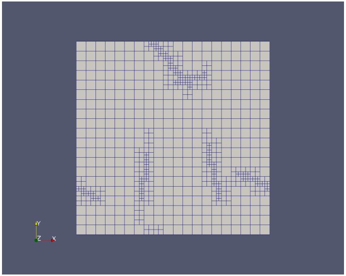

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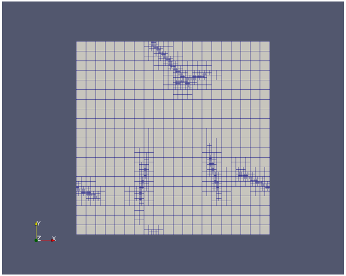

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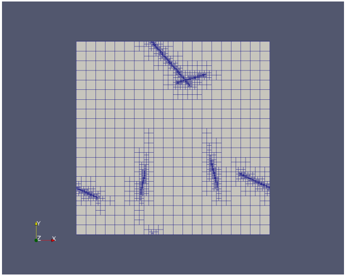

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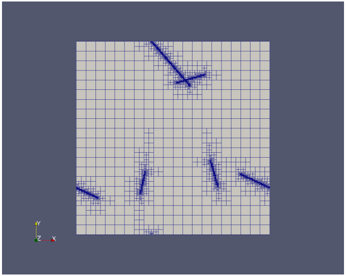

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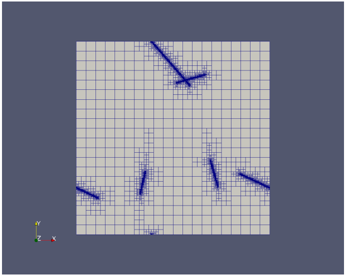

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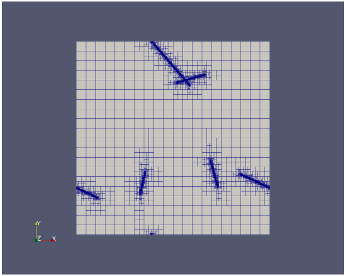


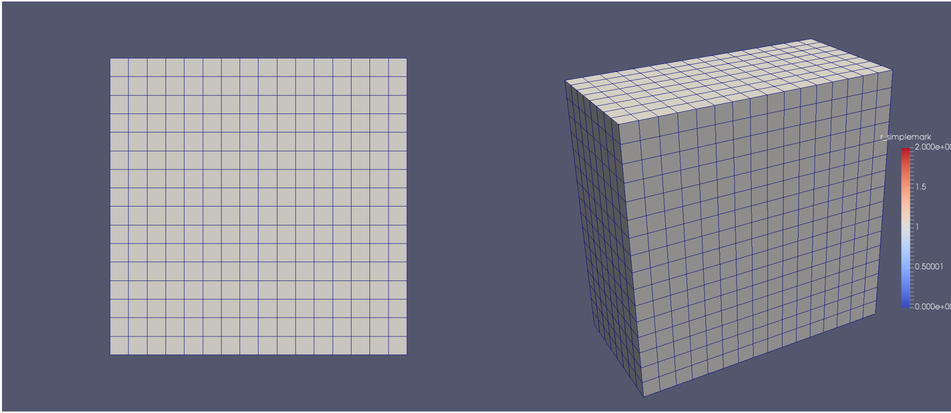
- New app Parrot in the FE framework MOOSE
- native adaptive mesh refinement
- native interface to parallel software PETSc

In particular,

- extension of MOOSE to work with complex type
- implementation of geometric multigrid solver for
    - adapted meshes
    - complex variables

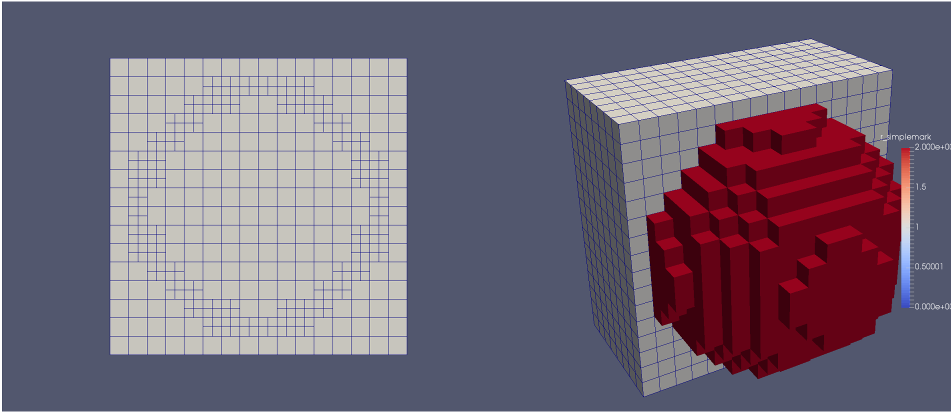
Validation of our approach

- spheric inclusion in a cube
- 3D test
- clamped normal displacements
- analytical solution provided by Pride et al. (2004)

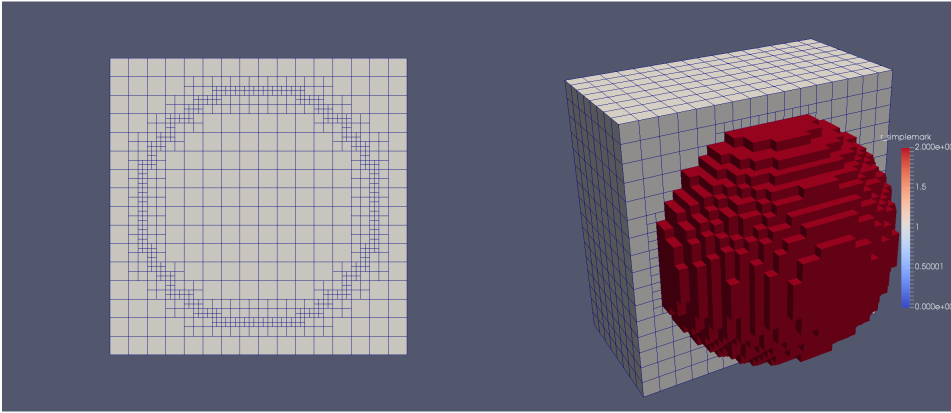


No. of nodes:	adaptive	uniform
	4913	4913

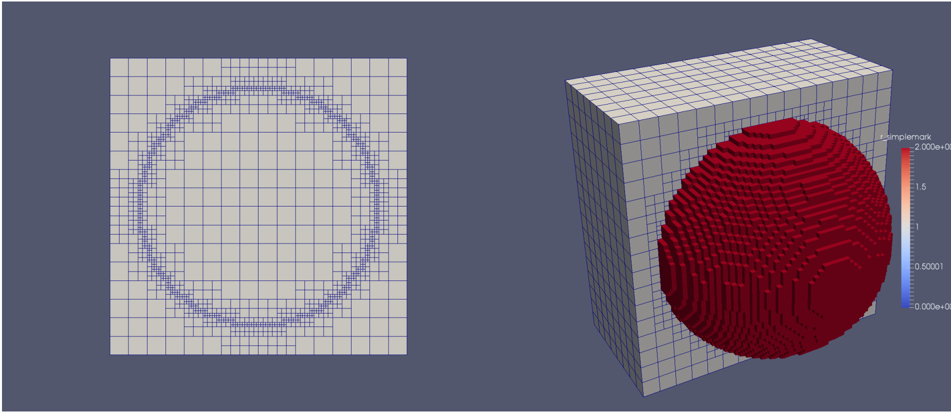
Spherically shaped gas inclusion



No. of nodes:	adaptive	uniform
	9528	35973

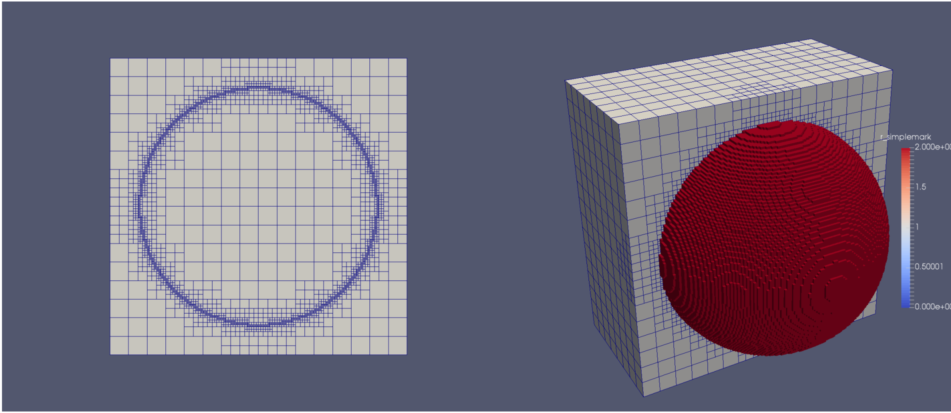


No. of nodes:	adaptive	uniform
	33944	274625



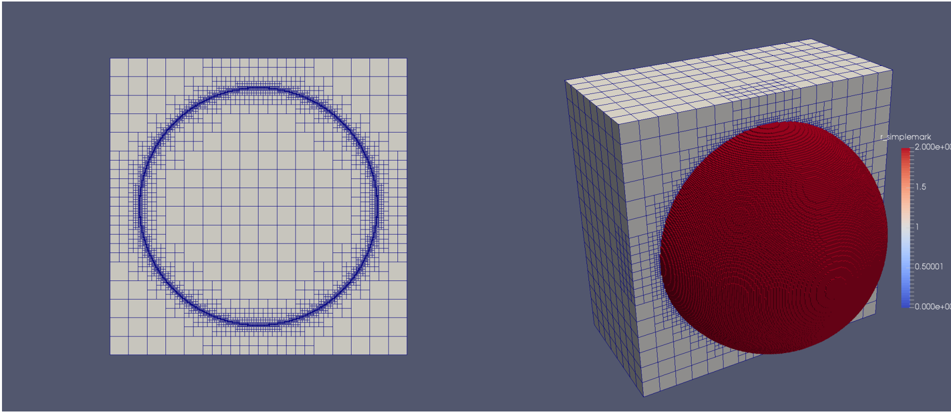
No. of nodes:	adaptive	uniform
	134464	2M

Spherically shaped gas inclusion



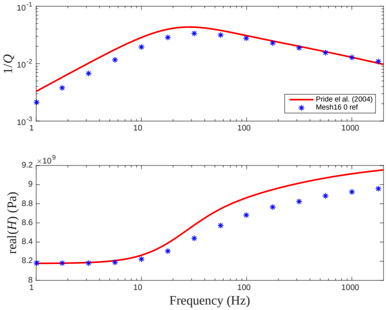
No. of nodes:	adaptive	uniform
	733279	16M

Spherically shaped gas inclusion



No. of nodes:	adaptive	uniform
	2.9M	135 M

Convergence to the analytical solution



- reproduces the curves over the all spectrum
- no difference between uniform and adaptive refinement
- adaptive algorithm needed for
  - dispersion at small frequencies
  - attenuation at large frequencies

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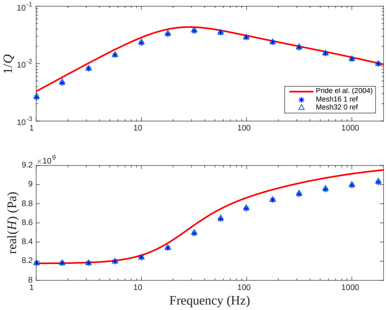

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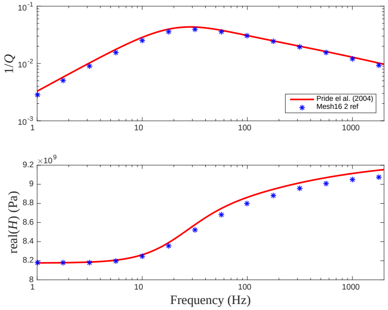

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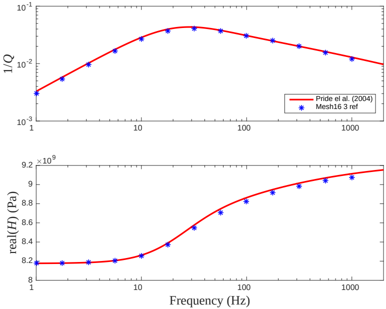

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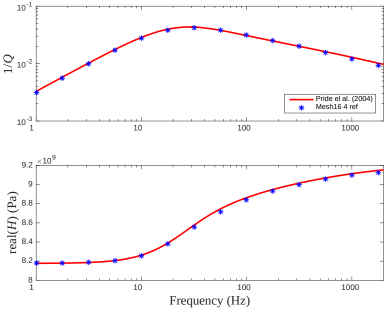

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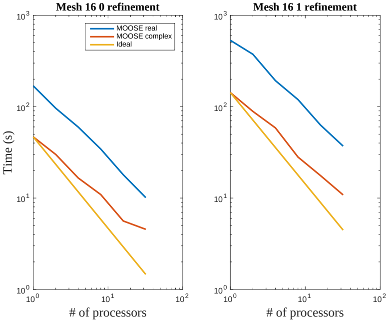
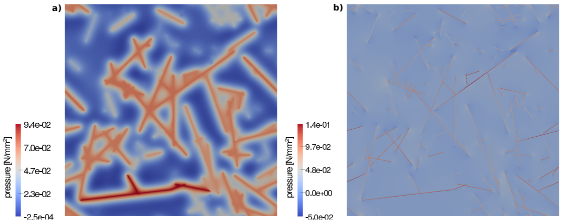
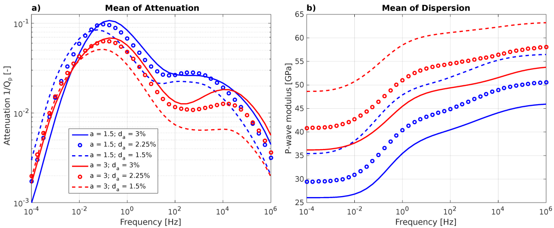


- better scaling for larger problems
- gain using complex MOOSE
    - from 2.2 to 3.6 for refinement 0
    - from 3.4 to 4.2 for refinement 1
- results with 4 refinements possible only with complex version



Real values of pressure and vertical real displacement at 10^{-1} and 10^{3} Hz



Parrot employed to compute

- displacement and pressure distributions
- dispersion and attenuation as functions of frequency
- mean value of 20 stochastic fracture networks
- see presentation by Eva Caspari and poster Jürg Hunziker

Conclusions

- Developed a novel software for fracture networks
- Conversion of FE framework MOOSE from Real to Complex
- AMR allows for stochastic simulations

Future works

- Multilevel Montecarlo to improve convergence of attenuation and modulus dispersion
- A-posteriori error estimate for complex poroelasticity

Thank you for your attention